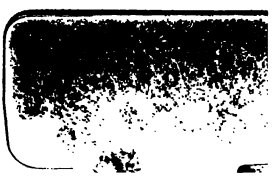

THE

RELATIONS AND DESCRIPTION

OF

FORMS,

ACCORDING TO THE PRINCIPLES OF

PESTALOZZI.

PART. I.

With four Copper-plate Engravings.

DUBLIN:

SOLD BY MARTIN KLENE, BOOKSELLER, COLLEGE-GREEN—THOMAS BOWES, 67, LOWER GARDINER-STREET—AND AT THE COMMITTEE-HOUSE FOR CHARITABLE SOCIETIES, 16, UPPER SACKVILLE-STREET.

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PREFACE.

THE branch of PESTALOZZI's Elementary Course of Instruction called Forms, (a small portion of the details of which are contained in the following pages,) is one, whose objects are so extensive and important, as to preclude the possibility of entering into a satisfactory explanation of it in this work. By its Inventor, it is defined to be "An artificial mode of exercising the eye of a Child to observe the forms and determine the dimensions of the various objects that meet his observation; and, also, of teaching his hand to represent them." And, it is plain, that under such a definition, the rudiments of Writing, Drawing, Mathematics, and indeed every Science which depends upon the faculty of sight, immediately arrange themselves.

It will appear by the above definition, that this Branch divides itself into two distinct parts.—1st. Observing the Forms and determining their Dimensions, which is called "The Relations of Forms." 2d.—Teaching the hand to describe them, which is called—"The Description of Forms." The Instruction of a Pupil may be said to begin with the Relations, in as much as the first step is to make him acquainted with a few common terms, such as Upright, Horizontal,

Slanting, Parallel, Straight, Curved and Crooked, In doing this a definition of these terms is, by no means required from a Child; but simply to ascertain by questions, whether he associates a correct idea with each of them. Thus you ask him—What do you understand by Round? This he is probably unable verbally to reply to, but the object of the question having been to convince him of his ignorance; that done, you immediately relieve him by asking him again, "to point out something that is Round?" If he replies by naming some object, such as a halfpenny, or a cart-wheel, that are round, you are certain that he associates a correct idea with the term, and your object is attained; and it still remains for a more advanced period to lead him to the discovery of those properties which characterize a Circle.

*Here the farther consideration of what may be termed "The Relations of Forms" pauses for a time to admit of some progress in describing them with the hand.—This ought to begin by *certain Gymnastic exercises, by which the Child learns how to use, 1st his whole arm from the shoulder joint.—2dly, His arm from the elbow. 3dly, His hand from the wrist.—4thly, Only the*

* As a universal course of gymnastic Exercises, whereby the Child is taught to observe the various movements of every joint of his body and limbs, and to practice them, forms a part of the general System, and will be attended to hereafter so more need be said of it in this place.

fingers. He is then made to take a little stick in his hand, and move it according to certain prescribed rules in every possible direction.

He may then proceed to describe on a large Slate, or blackened Board, placed nearly upright, the series of lessons in Curved lines contained in the following pages; and it will quickly strike an observant person, why Curved lines are more easy of execution, and consequently prior to Straight ones, inasmuch as the arm, when left to itself, naturally describes a part of a Circle; whereas the production of a Straight line requires the complex movement of more joints than one. The two series of Curved and of Straight lines may be considered as the first step in describing Forms, being the simple exercise of the hand. The next step is the exercise of the Eye in dividing (without being permitted to measure) Straight lines into various parts, such as halves, fourths, eighths, thirds, sixths, ninths, fifths, tenths, sevenths, &c. as preparatory to the 1st Table of the Relations of Forms, which immediately follows.

Having compleated these three Series, the Pupil will find himself furnished with certain elements of which the objects given in the Appendix are exclusively made up, and in the use of which one caution is highly necessary—namely, that it never was intended to pur

the mode of dictation through all the details of an elaborate drawing, but merely to carry it so far as it may be necessary to habituate the Pupil in his own mind, to compare the proportions of whatsoever object he is describing; beyond that, it must naturally appear as frivolous and absurd, as it will be found advantageous when kept within due limits.

It will also be observed that considerable progress towards the necessary Art of Writing will have been made by the acquirement of these elements, in as much as a general freedom of hand and accuracy of eye have prepared the pupil to observe and imitate the particular form presented in his copy, as also to discover his own errors in the execution of it. But to the details of this useful and necessary art exclusive attention shall be paid in a future publication.

It is hoped that Teachers will add many steps which arise out of the lessons here given; such as the applying Curves to all sorts of Slanting lines, and joining the several points of division in lines, divided into parts by Curved lines, and many similar, which would exceed the limits of this work. It is scarcely necessary to caution Teachers against attempting to follow in a MECHANICAL MANNER *the Questions and Exercises as they are given. Teachers who have not observation enough to vary their questions as occasion*

may point out, had better not undertake to give Instruction on this plan, as different means will be necessary with different dispositions and under different circumstances. It has been recorded of the Emperor CHARLES *V. that in his retirement he laboured for a considerable time in vain, to regulate two watches to keep time with one another. Finding the thing impracticable, he was struck with the absurdity of his own political career, in having endeavoured to give that uniformity of operation to the human mind, which he could not communicate to these little machines. Surely this little anecdote ought to prove a lesson to those Instructors who think to follow a beaten unalterable track in communicating any branch of knowledge to the human mind. They may overload the memory with crude materials, but these will remain useless and speedily moulder in the Stores, unless the reason be enabled to operate upon them with effect, and practically to apply them to their several purposes.— This latter will be found to be the object of this System in every branch of its advancement.*

The details of the Relations of Forms, together with the 1st Table, are given as the first attempt of PESTALOZZI, *to realize his views of teaching a Child to act by rule in observing the forms and dimensions of such objects as are presented to him; and he anxiously solicits such persons as approve of his views, rather to propose to themselves the same end, and taking* Nature

Q.—What then do you mean when you say a thing *(or a line)* is Upright?

Q.—Do you know what the words *HORIZONTAL*, or *FLAT*, mean?

Q.—Shew me something that is Horizontal?

Q.—Is the top of the Table Horizontal?

Q.—Shew me something else that is Horizontal?

Q.—Can a thing that is truly Horizontal slant in any direction?

Q. What is the meaning of the word Horizontal?

Q. Do you know why the word Horizontal is used to express flat or level things?

Q.—Did you ever observe in a plain country, or in sight of the sea, the appearance of a line where the sky and the earth, (or sea,) meet? This line is, in such cases, perfectly Flat, and is called the Horizon.

Q.—Can you now tell why the word Horizontal is used to signify Flat or Level?

Q.—Do you know what *SLANTING* means?

Q.—Shew me something that is Slanting?

Q.—Is the roof of the house Slanting?

Q.—Shew me something else that is Slanting?

Q.—Do both sides of the roof slant in the same direction?

Q.—How could you name the different directions in which they slant?

Q.—Which way does the side opposite your right hand slant?

Q.—Which way does the side opposite your left hand slant?

Q.—Can a thing be both Upright and Slanting?

Q.—Can a thing be both Horizontal and Slanting?

Q.—Is a Slanting line then either Horizontal or Upright?

Q.—What then is a Slanting line?

Q.—What do you mean by *STRAIGHT*?

Q.—Shew me something that is Straight?

Q.—Is that Rule Straight?

Q.—Shew me something else that is Straight?

Q.—Can a thing be Upright and Straight at the same time?

Q.—Can a thing be Upright without being Straight?

Q.—But can a thing be Straight without being Upright?

Q.—Can a thing be Horizontal and Straight at the same time?

Q.—Can a thing be Horizontal without being Straight?

Q.—Can a thing that Slants be Straight?

Q.—Can a thing Slant without being Straight?

Q.—Can a thing be Straight without Slanting?

Q.—What do you mean by *CURVED?*

Q.—Shew me something that is Curved?

Q.—Is the arch of a Bridge curved?

Q.—Shew me something else that is curved?

Q.—In what direction would you say the arch of a Bridge was curved?

Q.—It is bent upwards, *i. e.* the full part of the curve points upwards. But cannot a Curve line be bent in any other direction but upwards; for instance, what is the contrary of upwards?

Q.—Thus, if the ends of a Horizontal line were joined by a curve drawn above the line, how would you say the curve was bent?

Q.—If the ends of a Horizontal line were joined by a curve drawn below the line, how would you say the curve was bent?

Q.—But suppose the Curve line was drawn so as to join the ends of an Upright line, on the right hand side of it, how would you say the Curve was bent?

Q.—Why: which way does the swell of the curve lie?

Q.—Suppose the curve line was drawn to join the ends of the Upright line on the left side, which way would you say it was bent?

NOTE—These different directions of Curved Lines should be drawn on a Slate before the Pupil, and then the Questions asked respecting them; understanding by a Curved Line at first, a fourth part of a Circle.

Q.—What do mean by *CROOKED?*

Q.—What is the difference between Crooked and Straight?

Q.—Which is the shortest way from one place to another, a straight or a crooked path?

Q.—Suppose now two points on your Slate were two places, which would be the longest, a crooked or a straight line drawn to join them?

Q.—Is there any rule or regularity in a Crooked line?

Q.—Is there any rule or regularity in a Straight line?

Q.—What then is the difference between Crooked and Straight?

Q.—What is the difference between Crooked and Curved?

Q.—Is there any regularity in the bending of a Crooked line?

Q.—Is there any regularity in the bending of a Curved line?

Q.—What then is the difference between a Curved Line and a Crooked one?

Q.—You are right: "*A Curved line is bent regularly; a Crooked line is bent irregularly.*" Remember also, "*A Straight line is the shortest that can join two points, and cannot bend either way.*"

Q.—Do you know what *PARALLEL* means?

Q.—Shew me something that is Parallel?

Q.—Are the two lines formed by the two opposite edges of that slate the same distance from one another in every part?

Q.—If they are, they are called Parallel; what then does Parallel mean?

Q.—Are Horizontal lines Parallel?

Q.—Can Horizontal lines be other than Parallel?

Q.—Place that Slate so that two of its opposite edges may be Horizontal? Are they Paallel to one another?

Q.—Can Upright lines be Parallel?

Q.—Can they be other than Parallel?

Q.—Place two opposite edges of that Slate so that both may be Upright, but not both Parallel if you can?

Q.—Do you not then perceive that every Upright line must be Parallel to every other line that is Upright?

Q.—Can Slanting lines be Parallel?

Q.—Place two opposite edges of that Slate so that they may be both Slanting and Parallel?

Q.—Can two things Slant without being Parallel?

Q.—Shew me two lines that Slant without being the same distance from each other in every part.—Thus you see Slanting lines may be Parallel or may not, but Upright and Horizontal lines must be always Parallel to one another?

Q.—Can Curved lines be Parallel?

Q.—Can lines be Curved without being Parallel?

Q.—You are right: they may either be Parallel or not.

DESCRIPTION OF FORMS.

CHAP. I.

THE CHILD having been taught, as in the foregoing Questions, to observe various lines in the objects around him, and led to perceive the difference between Straight and Crooked—Crooked and Curved lines; to know which Straight lines are Upright, which Horizontal, and which Slanting. To perceive that Curved lines may be bent upwards, to either hand, or downwards, and to distinguish them from one another. He is now called to trace these lines with a piece of Chalk on a large Slate, or blackened Board.

SECT. I.—*PLATE I.*

Draw a Curved line bending upwards.—(*a*, *FIG.* 1)

Q.—Have you done it? What have you done?

NOTE—Each of these Figures should be repeated and dwelt upon until it can be performed with dexterity; and, moreover, should there be much awkwardness on the part of the Pupil in first essaying to draw them, it will be found expedient to make him lay down his Chalk and perform a little Gymnastic exercise, using his arm freely from the Shoulder joint only, and turning it round and round, first from Right to Left, then from Left to Right; after which he will be found to describe much better Curves on resuming his Chalk.

Having succeeded in drawing one tolerable Curved line, the orders are renewed as follows:—

Draw another Curved line below *(or above)* the first, bending upwards.—(*b*, FIG. 1.)

Q.—Have you done it? What have you done?

Below *(or above)* this second Curved line, draw a third Curved line, bending upwards.—(*c*, FIG. 1.)

Q.—Have you done it? What have you done?

Below *(or above)* this third Curved line, draw another Curved line, bending upwards. —(*d*, FIG. 1.)

Q.—Have you done it? What have you done?

Below *(or above)* this fourth Curved line, draw another Curved line, bending upwards.—(*e*, FIG. 1.)

Q.—Have you done it? What have you done?

You may now enquire: How many ends has one of these Curved lines? In a Curved line, bent upwards, how do you distinguish them one from another?

Q.—How are they called?

Q.—Which is the end towards your right hand or the right end?

Q.—Which is the end towards your left hand, or the left end?

SECT. II.

Draw a Curved line bending to the left.—(*a*, *FIG.* 2.)

Q.—Have you done it? What have you done?

Draw a second Curved line, bending to the left, at the right hand side of the first.—(*b*, *FIG.* 2.)

Q.—Have you done it? What have you done?

Draw a third Curved line, bending to the left, at the right hand side of the second.—(*c*, *FIG.* 2.)

Q.—Have you done it? What have you done?

Draw a fourth Curved line, bending to the left, at the right hand side of the third.—(*d*, *FIG.* 2.)

Q.—Have you done it? What have you done?

Draw a fifth Curved line bending to the left, at the right hand side of the fourth.—(*e*, *FIG.* 2.)

Q.—Have you done it? What have you done?

How many ends has a Curved line bent to the left?

How do you distinguish them one from another?

What do you call them?

Which is the Upper end? Which is the Lower end?

NOTE—Too much attention cannot be given by the Instructor to observe faults in the performance of each figure, as the principal object of these operations is to give manual facility. He should accustom the Pupil to repeat the command to himself while executing it, and also when it is compleated, in reply to the question "*What have you done?*" in order to be certain that he understands what he is doing.

SECT. III.

Draw a Curved line bending to the right.—(*a*, FIG. 3.)

Q.—Have you done it? What have you done?

Draw a second Curved line, bending to the right, on the left hand side of the first.—(*b*, FIG. 3.)

Q.—Have you done it? What have you done?

Draw a third Curved line, bending to the right, on the left hand side of the second.—(*c*, FIG. 3.)

Q.—Have you done it? What have you done?

Draw a fourth Curved line, bending to the right, on the left hand side of the third.—(*d*, FIG. 3.)

Q.—Have you done it? What have you done?

Draw a fifth Curved line, bending to the right, on the left hand side of the fourth.—(*e*, FIG. 3.)

Q.—Have you done it? What have you done?

How many ends has a Curved line bent to the right?

How do you distinguish them?

How are they called?

Which is the Upper end?

Which is the lower end?

SECT. IV.

Draw a Curved line bending downwards—*(a, FIG 4.)*

Q.—Have you done it? What have you done?

Draw a second Curved line, bending downwards, above *(or below)* the first.—*(b, FIG. 4.)*

Q.—Have you done it? What have you done?

Draw a third Curved line, bending downwards, above *(or below)* the second.—*(c, FIG. 4.)*

Q.—Have you done it? What have you done?

Draw a fourth Curved line, bending downwards, above *(or below)* the third.—*(d, FIG. 4.)*

Q. Have you done it? What have you done?

Draw a fifth Curved line, bending downwards, above *(or below)* the fourth.—*(e, FIG. 4.)*

Q.—Have you done it? What have you done?

Q.—How many ends has a Curved line, bending downwards?

Q.—How do you call them?

How do you distinguish them one from another?

Which is the end towards your right hand, or the right end?

Which is the left end?

SECT. V.

Draw a Curved line bending upwards.—*(a Fig. 5)*

Q.—Have you done it? What have you done?

From the right hand end of this line, and below it, draw a Curved line bending to the right.—*(b. Fig. 5)*

Q.—Have you done it? What have you done?

From the left hand end of the line bending upwards, draw a Curved line bending to the left.—*(c, Fig. 5.)*

Q.—Have you done it? What have you done?

Draw a Curved line bending downwards, joining the lower end of the Curved line bending to the left, to the lower end of the Curved line bending to the right.—*(d, Fig. 5.)*

Q.—Have you done it? What have you done?

Q.—What is this Figure called?

Q.—What is the point in the middle of it called?

Q.—Could you draw from the point in the middle, which is called the *Centre*, to the Curved line of the Circle two Straight lines that would be of different lengths?

Q.—What lengths then are all the Straight lines you can draw from the Center to the Curved line of the Circle, when compared with one another.

SECT. VI.

Mark a point on the left hand side of your Slate. Beginning at that point draw, towards your right hand, a Curved line bending upwards—*(a, Fig. 6.)*

Q.—Have you done it? What have you done?

Mark a point on the right hand side of your Slate. Beginning at that point draw, towards your left hand, a Curved line bending upwards.—*(b, Fig. 6.)*

Q.—Have you done it? What have you done?

Mark a point on the left side of your Slate. Beginning at that point draw, towards your right hand, a Curved line bendingdownwards.—*(c, Fig. 6.)*

Q.—Have you done it? What have you done?

Mark a point at the right hand side of your Slate. Beginning at that point draw, towards your left hand, a Curved line bending downwards.

Q.—Have you done it? What have you done?

SECT. VII.

From a point at the top of your Slate draw, towards the bottom, a curved line bending to the left.—*a, Fig. 7.*

Q.—Have you done it? What have you done?

From a point near the bottom of your Slate draw, towards the top, a Curved line bending to the left. —*(b, Fig. 7.)*

Q.—Have you done it? What have you done?

From a point near the top of your Slate draw, towards the bottom, a Curved line bending to the right. —*(c, Fig. 7)*

Q.—Have you done it? What have you done?

From a point near the bottom of your Slate, draw towards the top, a Curved line bending to the right. —*(d, Fig. 7.)*

Q.—Have you done it? What have you done?

NOTE.—You may now, for Exercise vary the first four lessons, by directing the Child to draw the lines alternately, beginning at the right or left, at the top or bottom of the Slate. This will form his hand to evenness of execution, and to the precise following of orders.

SECT. VIII.

From a point in the middle of the Slate draw, towards your right hand, a Curved line bending upwards.—*(a, Fig. 8.)*

Q.—Have you done it? What have you done?

From the same point draw, towards your left hand, a Curved line bending upwards.

Q.—Have you done it? What have you done?

From a point on the left side of your Slate draw, towards your right, a Curved line bending upwards.—From the same point draw towards the bottom of the Slate a Curved line bending to the right hand.—*(b, Fig. 8.)*

Q.—Have you done it? What have you done?

From a point near the lower part of your Slate, equally distant from each side, draw, towards the top, a Curved line bending to the left hand. From the same point draw, towards your left hand, a Curved line bending downwards.—*(c, Fig. 8.)*

Q.—Have you done it? What have you done?

SECT. IX.

From a point in the upper part of the right side of the Slate draw, towards your left hand, a Curved line bending upwards. From the same point draw, towards the bottom of the Slate, a Curved line bending to the left.—*(a, Fig. 9.)*

Q.—Have you done it? What have you done?

From a point in the lower part of the left side of the slate draw, towards your right hand, a Curved line bending downwards. From the same point draw, towards the top of the Slate, a Curved line bending to the right.—(*b*, *Fig.* 9.)

Q.—Have you done it? What have you done?

Near the upper part of the Slate, draw a Curved line bending downwards. From the right end of that line, draw, towards the bottom of the Slate, a Curved line bending to the left. From the left end of the line bending downwards draw, towards the bottom of the Slate a Curved line bending to the right hand.—(*c*, *Fig.* 9.)

Q.—Have you done it? What have you done?

Connect the lower ends of the Curved lines bending to the right and left hands by a Curved line bending upwards.

NOTE.—The practice on this last Figure may be much varied, according as you begin the Figure by any of the Curved lines which compose it, and according as you begin each Curve at either end.

SECT. X.

From a point in the middle of the Slate draw, towards your right hand, a Curved line bending upwards. From the same point draw, towards your left hand, a Curved line bending downwards. (*a*, *Fig.* 10)

Q.—Have you done it? What have you done?

From a point in the middle of the Slate draw, towards your right hand, a Curved line bending downwards. From the same point draw, towards your left hand, a Curved line bending upwards.—(*b*, *Fig.* 10.)

Q.—Have you done it? What have you done?

From a point in the middle of your Slate draw, towards the top, a Curved line bending to the left.—From the same point draw, towards the bottom, a Curved line bending to the right.—(*c*, *Fig.* 10)

Q.—Have you done it? What have you done?

From a point in the middle of the Slate draw, towards the top, a Curved line bending to the right.—From the same point draw, towards the bottom of the Slate, a Curved line bending to the left. (*d*, *Fig.* 10)

Q.—Have you done it? What have you done?

NOTE.—For practise these Figures may then be drawn beginning at the end instead of the middle; for instance, *Fig.* 10—Draw a Curved line bending downwards, and then, at the right end of it draw, towards the right hand, a Curved line bending upwards; and, perhaps, when the Child's hand acquires expertness and freedom, he may be called on to execute the whole at once.

SECT. XI.

Draw a Curved line bending to the right, beginning at either the top or bottom of the Slate, and towards either the bottom or top. From the lower or upper end draw to the other end, a Curved line bending to the left.—(*a*, *Fig.* 11.)

Beginning at either the right or left side of the slate, draw either to the left or to the right, a Curved line bending upwards. From either the right or left end of the line, and to the other end, draw a Curved line bending downwards.—(*b*, *Fig.* 11.)

Towards the top of your Slate draw a Curved line bending downwards. From the right end of this line draw, towards the bottom of the Slate, a Curved line bending to the right. From the left end of the line bending downwards, draw, towards the bottom of the Slate, a Curved line bending to the left. From the lower end of the Curved line, bending to the left, to the lower end of the Curved line, bending to the right, draw a Curved line bending Upwards.—(*c*, *Fig.* 11.)

Remembering, invariably, to ask the Questions—Have you done it? What have you done?

SECT XII.

From a point in the centre of the Slate draw, towards the top a Curved line bending to the right.—(*a*, *Fig.* 12.)

From the same point draw, towards the top of the Slate a Curved line bending to the left.

From the same point draw towards the right side of the Slate a Curved line bending upwards.

From the same point draw, towards the right side of the Slate, a Curved line bending downwards.

From the same point draw, towards the bottom of the Slate, a Curved line bending to the right.

From the same point draw, towards the bottom of the Slate, a Curved line bending to the left.

From the same point draw, towards the left hand, a Curved line bending upwards.

From the same point draw, towards the left hand, a Curved line bending downwards.

Remembering, as before, to ask the Questions—Have you done it? What have you done?

NOTE.—When the Pupil has practised Curved Lines of an uniform thickness throughout, he ought to draw them in all directions, full in the middle and fine at the points, such as are given in Figs. 13 and 14—Plate I.

CHAP. II.

HAVING thus learnt to be tolerably expert in drawing Curved lines in any of the four principal directions, and in combining these in any way that may be directed, the Pupil's next step is the describing of *Straight Lines*, which have also been shewn to admit of Four principal directions, viz. *Horizontal, Upright, Slanting to the Right, and Slanting to the Left.*

SECT. I.—PLATE II.

Draw a Horizontal Straight Line.—*a*, *Fig.* 1.

Q.—Have you done it? What have you done?

Above the first, draw another Horizontal line equal and parallel to the first.—*b*, *Fig.* 1.

Q.—Have you done it? What have you done?

Above the second, draw another Horizontal line equal and Parallel to the first.—*c*, *Fig.* 1.

Q.—Have you done it? What have you done?

Above the third, draw another Horizontal line equal and parallel to the first.—*d*, *Fig.* 1.

Q.—Have you done it? What have you done?

Above the fourth, draw another Horizontal line equal and parallel to the first.—*e*, *Fig.* 1.

Q.—Have you done it? What have you done?

How do you distinguish the ends of a Horizontal line one from another?

Which is the right end? Which is the left end?

SECT. II.

Draw an Upright Straight line.—*a*, *Fig.* 2.

Draw another Upright Straight line to the right, of the first.—*b*, *Fig.* 2.

Draw a third Upright Straight line to the right, of the second.—*c*, *Fig.* 2.

Draw a fourth Upright Straight line to the right, of the third.—*d*, *Fig.* 2.

Draw a fifth Upright Straight Line to the right, of the fourth.—*e*, *Fig.* 2.

How do you distinguish the ends of an Upright line one from another.

Which is the upper end? Which is the lower end?

Still remembering to ask the questions—Have you done it? What have you done? after each operation.

SECT. III.

Draw a Straight line slanting downwards from left to right.—*a, Fig.* 3.

On the left side of this line draw another Slanting line equal and parallel to the first.—*b, Fig.* 3.

On the left side of the second line, draw another Slanting line equal and parallel to the first—*c, Fig* 3.

On the left side of the third line draw another Slanting line equal and parallel to the first.—*d, Fig.* 3.

On the left side of the fourth line draw another Slanting line equal and parallel to the first—*e, Fig.*3.

How do you distinguish the ends of a Slanting line one from another?

Which is the upper end of a line slanting downwards from left to right?

Which is the lower end of it?

SECT. IV.

Draw a Straight line Slanting downwards from right to left.—*a Fig.* 4.

On the right side of this line draw another Slanting line equal and parallel to the first.—*b*, *Fig.* 4.

On the right side of the second line draw another Slanting line equal and parallel to the first.—*c*, *Fig.* 3.

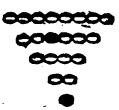
On the right side of the third line draw another Slanting line equal and parallel to the first.—*d*, *Fig.* 3.

On the right side of the fourth line draw another Slanting line equal and parallel to the first.—*e*, *Fig.* 3.

How do you distinguish the ends of a Slanting line one from another?

Which is the upper end of a line Slanting downwards from right to left?

Which is the lower end of it?



CHAP. III.

As soon as the Pupil can describe Straight Lines with tolerable facility, according to the preceding chapter, the next Forms to which his attention should be directed are *Angles*, to which the following questions are introductory.

QUESTIONS.

Do you know what the point is called where one Straight Line meets another?

Do you perceive that when two lines meet there is a space *within*, and a space *without* them?

Which do you call *within* the two lines that meet; the space on that side the lines where they are nearest together, or where they are farthest asunder?

The space then that lies within two lines that meet, and more especially the point where they meet, is called *An Angle*. Shew me two things in the room that make an Angle?

Do the top and the side of the window make an Angle where they meet?

What then is an Angle?

Could you shew me two other things in the room that meet and form an Angle?

There are also different kinds of Angles. When an Upright line meets a Horizontal line, what kind of an Angle is it called?

Shew me something in the room in which an Angle is formed by an Upright line meeting a Horizontal one?

Is the side of the Fire-place an Upright line?

Is the top of it a Horizontal line?

Do they form an Angle?

Is it called a right Angle?

Could you shew me two other lines about the room which meet, and form a Right Angle?

Do you observe where the two sides of the Roof meet in the Ridge?

Do they form an Angle?

Is it a Right Angle?

Is it sharper or blunter than a Right Angle?

True, it is sharper than a Right Angle. An Angle sharper, or less than a Right Angle, is called a Sharp, or Acute Angle. Could you shew me two other lines that meet in an Acute Angle?

Do you observe where the Roof of the House meets the Wall? Do they form an Angle?

Is it a Right Angle?

Is it sharper or blunter than a Right Angle?

Yes, it is Blunter than a Right Angle. An Angle blunter, or greater than a Right Angle, is called a Blunt, or Obtuse Angle. Could you shew me two other lines that meet and form an Obtuse Angle?

What then do you mean by a Right Angle?

What do you mean by an Acute Angle?

What do you mean by an Obtuse Angle?

SECT. I.

Draw an Upright line.—*a Fig. 5.*

From the lower extremity of it, draw, towards the right, a Horizontal line.

What sort of an Angle do these lines make?

Draw a Horizontal line.—*b, Fig. 5.*

From the left extremity of it draw, towards your left, a Straight line slanting upwards from right to left.

What sort of an Angle do these two lines make one with another?

Why is it an Obtuse or Blunt Angle?

Draw a Horizontal line.—*c*, *Fig.* 5.

From the left extremity of it draw, towards your right, a Straight line slanting upwards from left to right.

What sort of an Angle do these two lines make one with another?

Why is it an Acute or Blunt Angle?

SECT. II.

Draw a Horizontal line.—*a Fig.* 6.

From the right end of it draw an Upright line below it.—Or thus:

Draw an Upright line.

From the upper end of it draw, towards the left, a Horizontal line.

Draw an Upright line.—*b*, *Fig.* 6.

From the upper end of it draw, towards the right, a Horizontal line.—Or thus:—Draw a Horizontal line.

From the left end of it, and below it, draw an *Upright* line.

SECT. III.

Draw an Upright line.—*a*, *Fig.* 7.

From the upper extremity of it draw, towards the right, a Horizontal line.

From the upper extremity of it draw, towards the left, a Horizontal line.

Draw an Upright line.—*b*, *Fig.* 7.

From the lower extremity of it draw, towards the right, a Horizontal line.

From the lower extremity of it draw, towards the left, a Horizontal line.

SECT. IV.

Draw a Horizontal line.—*a*, *Fig.* 8.

From the right end of it, draw an Upright line above it.

From the right end of it, draw an Upright line below it.

From the right end of it draw, also, another Horizontal line towards the right.

Draw a Horizontal line.—*a*, *Fig.* 9.

From the right end, and above it, draw an Upright line equal to it.

From the left end of it, and above it, draw another Upright line equal to it.

Join the upper ends of these Upright lines by a Horizontal line.

CHAP. IV.

THE Pupil being familiarized with the three kinds of Angles in the preceding Chapter should now be directed to observe the differences of magnitude in Horizontal and Upright Straight Lines, and to divide them, by his eye, into various parts.

SECT. I—PLATE II.

Draw a Horizontal Straight Line.—*a*, *Fig.* 10.

Below it draw another Horizontal line less than the first.

Below these, draw another Horizontal line, greater than the second and equal to the first.

Below these draw another Horizontal line less than the first and equal to the second.

Below these draw another Horizontal line, greater than the fourth and equal to the first and third.

How many greater lines have you drawn?

Are these three lines equal or unequal?

How many lesser lines have you drawn?

Are these two lesser lines equal or unequal?

Name which lines are greater and which less.

Draw a Horizontal Straight line.—*b*, *Fig.* 10.

Below it draw another, less than the first.

Below these draw another Horizontal line, equal to the second and less than the first.

Below these draw another Horizontal line less than the second and third and equal to the first.

How many greater lines are there here?

Are they equal or unequal?

How many lesser lines have you here?

Are they equal or unequal?

Name the greater lines that are equal to one another.

Name the lesser lines that are equal to one another.

Draw a Horizontal Straight line.—*c*, *Fig.* 10.

Draw another Horizontal line, below and greater than it.

Draw another Horizontal line below the second, greater than both first and second.

Below these draw a fourth Horizontal line equal to the third.

Below these, draw a fifth Horizontal line, equal to the second.

Below these, draw a sixth Horizontal line equal to the first.

How many sizes of lines are there here?

How many lines of each magnitude?

Name the lines that are equal to one another.

Draw a Horizontal Straight line.—*d*, *Fig.* 10.

Below this, draw another Horizontal line equal to the first.

Below these, draw another Horizontal line, equal to the second.

Below these, draw another Horizontal line, equal to the third.

How many sizes of lines are there here? Are they equal or unequal? Name the lines that are equal?

SECT. II.

Draw an Upright Straight line.—*a*, *Fig.* 11.

To the left of it draw another Upright line, less than the first.

To the left of these draw another Upright line, greater than the second and equal to the first.

To the left of these draw another Upright line less than the first and equal to the second.

To the left of these, draw another Upright line, greater than the fourth and equal to the first and third

How many greater lines have you drawn ?

Are these three lines equal or unequal ?

How many lesser lines have you drawn ?

Are these two lesser lines equal or unequal ?

Name which lines are greater and which less,

Draw an Upright Straight line.—*b*, *Fig.* 11.

To the left of this draw another, less than the first.

To the left of these draw another Upright line, equal to the second and less than the first.

To the left of these draw another Upright line, greater than the second and third & equal to the first.

How many greater lines are there here ?

Are they equal or unequal?

Name the greater lines that are equal to one another.

Name the lesser lines that are equal to one another.

Draw an Upright Straight line.—*c*, *Fig.* 11.

Draw another Upright line to the left of the first, and greater than it.

Draw another Upright line to the left of the second, and greater than both first and second.

To the left of these draw a fourth Upright line equal to the third.

To the left of these draw a fifth Upright line equal to the second.

To the left of these draw a sixth Upright line equal to the first.

How many sizes of lines are there here?

How many lines of each magnitude?

Name the lines that are equal to one another.

Draw an Upright Straight line.—*d*, *Fig.* 11.

To the left of this draw another Upright line equal to the first.

To the left of these draw another Upright line *equal to the* second.

Below these draw another Upright line, equal to the third.

How many sizes, or magnitudes of lines are there here? Are they equal or unequal?

Name the lines that are equal?

SECT. III.

Draw a Horizontal Straight line.—*a*, *Fig.* 12.

Below it, draw another Horizontal line equal to it. *b*, *Fig.* 12.

Divide the second line, by one point, into two equal parts.

NOTE.—As the parts will, probably, be unequal at the first attempt, the Instructor should lead the Pupil to discover their inequality by Questions. Thus: "Are those parts equal?" "Which is the greatest?" "Mark off as much of the greater as you think it exceeds the lesser." "Rub out your former point."—Thus leading him on, in every instance, to perceive and correct his own errors.

When a line is divided into two equal parts what are they called. What then are the parts of the line you have just divided?

How many of them are there?

Which is the first half? which is the second half?

Below the second Horizontal Straight line, draw a third, equal to the first.—*c*, *Fig.* 12.

Divide this 3d line by one point into two equal part

Divide the first half of it, also, by one point into two equal parts.

Divide the second half of it also, by one point into two equal parts.

Into how many parts have you now divided the whole line?

By how many points?

When a line is divided into four equal parts what are each of them called?

Where is the first 4th situated?

Where is the second 4th situated?

Where is the third?

Where is the fourth?

NOTE.—For the manner of naming the parts of a divided line, See the "Relations of Forms," 4th Section, Page 5.

Below the third Horizontal Straight line draw a fourth Horizontal line, equal to the first.—*d*, *Fig.* 12.

Divide it by one point into two equal parts.

Divide each half, by one point, into two equal parts.

Into how many parts now is the whole line divided?

By how many points?

When a line is divided into eight equal parts what is each of them called?

Which is the first eighth? which is the second? which is the third? which is the fourth? which is the fifth? which is the sixth? which is the seventh?—which is the eighth?

Draw another Horizontal Straight line below the fourth equal to the first.—*e*, *Fig.* 12.

Divide it by two points into three equal parts.

When a line is divided into three equal parts what is each of them called?

Where is the first third situated? Where is the second? Where is the third?

Draw another Horizontal Straight line below the fifth, equal to the first.—*f*, *Fig.* 12.

Divide it by two points into three equal parts.

Divide each of these thirds by one point into two equal parts.

Into how many parts is the whole line divided.

By how many points?

When a line is divided into six equal parts what is each of them called?

Where is the first sixth situated? where is the second? where is the third? where is the fourth? where is the fifth? where is the sixth?

Draw another Horizontal Straight line below the sixth, equal to the first.—*g*, *Fig.* 12.

Divide it by two points into three equal parts.

Divide each of these thirds by two points into three equal parts.

Into how many parts is the whole line now divided?

By how many points?

When a line is divided into nine parts what is each of them called?

Which is the first ninth? which is the second? which is the third? which is the fourth? which is the fifth? which is the sixth? which is the seventh? which is the eighth? which is the ninth?

Draw another Horizontal Straight line below the seventh, equal to the first.—*h*, *Fig.* 12.

Divide it by four points into five equal parts.

When a line is divided into five equal parts what is each of them called? Where is the first fifth situated? Where is the second? Where is the third? Where is the fourth? Where is the fifth?

Draw another Horizontal Straight Line below the eighth, equal to the first.—*i*, *Fig.* 12.

Divide this line by four points into five equal parts.

Divide each of these fifths by one point into two equal parts.

Into how many fifths is the whole line now divided?

By how many points?

When a line is divided into ten equal parts what is each of them called?

Where is the first tenth situated? where is the second? where is the third? where is the fourth?—where is the fifth? where is the sixth? where is the seventh? where is the eighth? where is the ninth? where is the tenth?

Draw another Horizontal Straight line below the ninth, equal to the first.—*k*, *Fig.* 12.

Divive this line by six points into seven equal parts.

When a line is divided into seven equal parts what is each of them called?

Where is the first seventh situated? where is the second? where is the third? where is the fourth? where is the fifth? where is the sixth? where is the seventh?

How do you divide a line into halves?

How do you divide a line into fourths?
What do you do first? what afterwards?
How do you divide a line into eighths?
What do you do first? what next? what lastly?
How do you divide a line into thirds?
How do you divide a line into sixths?
What do you do first? what afterwards?
How do you divide a line into ninths?
What do you do first? what afterwards?
How do you divide a line into fifths?
How do you divide a line into tenths?
What do you do first? what afterwards?
How do you divide a line into sevenths?

NOTE.—The lessons of this Section should be carefully repeated in the division of Upright Straight Lines, referring to *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *k*, *Fig.* 13, altho' it is deemed unnecessary to repeat them here in print; and it should be invariably remembered throughout, after each separate operation, to ask the questions "Have you done it?" "What have you done?"

SECT. IV.

Near the left side of your Slate draw a short Horizontal line.—*Fig.* 14.

Below this line draw another equal to it.

To the right extremity of the second add a Horizontal line equal to the first.

Below these draw a Horizontal line equal to the whole of the second.

From the right extremity of the third draw, towards the right, a Horizontal line equal to the second.

Below these draw a Horizontal line equal to the whole of the third.

From the right extremity of this line draw, towards the right, a Horizontal line equal to the whole of the third.

How many times the first of these lines is the second equal to?

How many times the second is the third equal to?

How many times the first then is the third equal to?

How many times the third is the fourth equal to?

How many times the second then is the fourth equal to?

And how many times the first is the fourth equal to?

NOTE.—The Exercise of this Section should be repeated upon Upright Straight Lines, as in *Fig.* 15, and any questions added that suggest themselves to the Instructor, towards making the Pupil fully comprehend every relation between their magnitudes. Remembering as before, the constant use of the questions—"Have you done it?" "What have you done?" after each operation.

SECT. V.

Towards the upper part of the Slate draw a short Horizontal Straight line.—*Fig.* 16.

Below this first line draw a Horizontal line equal to it.

To the right extremity of this second, add a Horizontal line equalto the first.

To the left extremity also of the second add a Horizontal line equal to the first.

Below these, draw a third Horizontal line equal to the whole of the second.

To the right extremity of this line add a part equal to the first.

To the left extremity of this line also add a part equal to the first.

Below these, draw a fourth Horizontal line equal to the whole of the third.

To the right extremity of it add a part equal to the first.

To the left extremity of it add a part equal to the first.

Below the fourth line draw a fifth Horizontal line equal to the third, and placed immediately under it.

Below the fifth draw a sixth Horizontal line equal to the second, and placed immediately under it.

Below the sixth draw a seventh Horizontal line equal to the first, and placed immediately under it.

Mark all the lines, with points, into parts equal to the first line.

How many times the first then, is the second equal to ?

How much does the second exceed the first ?

How many times the first line is the third line equal to ?

How much then does the third line exceed the first ?

How much does the third line exceed the second ?

How many times the first line is the fourth line equal to ?

How much then does the fourth line exceed the first ?

How much does the fourth line exceed the second?

How much does the fourth line exceed the third ?

Is there any line to which the fifth is equal ?

Is there any line to which the sixth is equal ?

Is there any line to which the seventh is equal ?

NOTE.—This Exercise should also be repeated with Upright Straight Lines, as in *Fig.* 17, and any additional questions may be asked by the teacher, respecting the various relations of these magnitudes.

CHAP. V.

THE Pupil having learned, in the foregoing Exercises, accurately to draw Horizontal and Upright Straight Lines, and to divide them, exactly, into any number of parts, as far as ten, may now proceed to describe the Forms contained in the First Table, preparatory to considering their Relations.

SECT. I.

(See Nos. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, Relations of Forms, Table I.)

On the left hand side, near the Upper part of your Slate, draw a short Horizontal Straight line.

Below it draw another Horizontal line equal to twice the first.

Divide the second line by one point into two equal parts.

Below the second, draw a third Horizontal line equal to three times the first.

Divide it by two points into three equal parts.

Below the third Horizontal line draw a fourth, equal to four times the first.

Divide it by three points into four equal parts.

Below the fourth Horizontal line draw a fifth, equal to five times the first.

Divide it by four points in o five equal parts.

Below the fifth, draw a sixth Horizontal line, equal to six times the first.

Divide it by five points into six equal parts.

Below the sixth, draw a seventh Horizontal line equal to seven times the first.

Divide it by six points into seven equal parts.

Below the seventh, draw an eighth Horizontal line equal to eight times the first.

Divide it by seven points into eight equal parts.

Below the eighth, draw a ninth Horizontal line equal to nine times the first.

Divide it by eight points into nine equal parts.

Below the ninth draw a tenth Horizontal line, equal to ten times the first.

Divide it by nine points into ten equal parts.

This Exercise should then be repeated with Upright Straight lines, as in *Nos.* 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, of the First Table of the Relations of Forms.

The Relations of these Lines and their parts, should then be considered as given in the Six First Sections of the *Relations of Forms*, in applying which, Questions like the following may be used :

(See Relations of Forms, Sect. 1, Page 1.)

Q.—Name the Horizontal lines for me.

Q.—Which is the first Horizontal line?

NOTE.—In naming Horizontal lines it may be well, as a general rule, always to call the uppermost line the first, and in like manner in Upright lines, always to call the line next the left hand the first.

Q.—Which is the second Horizontal line?

Q.—Which is the third Horizontal line?

Q.—Which is the fourth? which is the Fifth? which is the Sixth? which is the Seventh? which is the Eighth? which is the Ninth? and which is the Tenth?

Q.—Which is the First Upright Straight Line?

Q.—Which is the Second Upright line? which is the Third? which is the Fourth? which is the Fifth? which is the Sixth? which is the Seventh? which is the Eighth? which is the Ninth? and which is the Tenth?

(See Relations of Forms, Sect. 2, Page 2.)

Q.—Which is the longest, the First, or the Second Horizontal line?

Q.—What lines are longer than the Fourth horizontal line?

Q—What lines are shorter than the Fourth horizontal line?

Q—Which is the longest, the First or the Second Upright line?

Q.—What lines are shorter than the Seventh Upright line?

NOTE.—Questions like the above may be repeated until the Pupils are perfectly acquainted with the names and relative magnitudes of all the lines, both Horizontal and Upright, in the parts of the Table they have already described.

(See Relations of Forms, Sect. 5, Page 5.)

Q.—Is the First horizontal line divided?

Q—Is the Second horizontal line divided?

Q.—By how many points?

Q.—Into how many parts?

Q.—Is the First Upright line divided?

Q.—Is the Second Upright line divided?

Q.—By how many points? Into how many parts?

Q.—How many points are necessary to divide a line into seven equal parts?

Q.—When a line is divided by four points into how many parts is it cut?

(See Relations of Forms, Sect. 4, Page 31).

Q.—When a Horizontal line is divided into two equal parts, what is each of them called?

Q.—Where is the First half situated?

Q.—Where is the Second half situated?

Q.—How many halves do the whole line contain?

Q.—When a horizontal line is divided into three equal parts, what is each of them called?

Q.—Where is the First third situated?

Q.—Where is the Second third situated?

Q.—Where is the Third third situated?

Q—How many Thirds does the whole line contain?

And so on with the various parts of the other divided lines both Upright and Horizontal.

NOTE.—When a Horizontal line is divided it is usual to begin numerically to name the parts from left to right, and of Upright Straight lines from top to bottom.

How many ways can you take one-fourth of the Fourth line?

Q.—Name where each of them is situated.

Q.—How many ways can you take two-fourths of the Fourth line?

Q.—Where are they respectively situated?

Q.—How many ways can you take three-fourths o f he fourth line ?

Q.—Name their situation ?

Q.—How many ways can you take four-fourths of the fourth line ?

Q.—Where is it situated ?

NOTE.—The manner of expressing these compound partitions of the several lines is given in the Fourth Section of the Relations of Forms, and should be repeated at full length upon every line, both Upright and Horizontal.

(See Relations of Forms, Sect. 5, Page 13.)

Q.—To what part of the second horizontal line is the first equal ?

Q.—To what part of the third, is half the second equal ?

Q.—To what part of the third is the whole of the second equal ?

Q.—To what part of the fourth line is two-thirds of the third equal ?

Q.—To what line is one-fourth part of the fourth line equal ?

Q.—To what line is three-fifths of the fifth line equal ?

Q.—To what line is four-sevenths of the seventh line equal ?

Q.—To what line is five-eights of the eighth line equal?

Q.—To what line is six-ninths of the ninth line equal?

Q.—To what line is seven tenths of the tenth line equal?

NOTE.—The whole substance of the 5th and 6th Sections, of the Relations of Forms, should be repeated in Questions like the above, until every possible relation which the parts of one line bear to the parts of another, be accurately observed and imprinted, without confusion, on the mind of the Pupil.

CHAP. VI.

SECT. I.

(See First Table, Fig. 21.)

Q.—Draw a horizontal straight line.

Q.—Draw another horizontal line below it, parallel to the first.

Q.—Why is the first of these lines parallel to the second?

(See First Table, Fig. 22.)

Q.—Draw an Upright Straight line.

Q.—Draw a second upright straight line to the right hand side of the first.

Q.—Why are these lines parallel to one another?

SECT. II.

(See First Table, Fig. 23.)

Q.—Draw a Horizontal Straight line.

Q.—From the right extremity of it draw an Upright straight line below it.

Q.—What sort of an angle do these two lines make one with another?

Q.—How would you name the sides of this angle?

Q.—What line forms the horizontal side of this angle?

Q.—What line forms the upright side of this angle?

(See First Table, Fig. 24.)

Q.—Draw a Horizontal Straight line.

Q.—Divide it by one point into two equal parts.

Q.—Draw an Upright line from the point of division below the horizontal line.

Q.—What angles are formed by these lines?

Q.—By how many lines are these two angles formed?

Q.—Which would you call the first angle?

Q.—What forms the horizontal side of the first angle?

Q.—What forms the Upright side of the first angle?

Q.—Which is the second right angle?

Q.—What forms the horizontal side of the second right angle?

Q.—What forms the upright side of the second right angle?

Q.—When the same line forms a side of two angles at the same extremity of it, what are those angles called?

Q.—What is the point of the two angles called?

NOTE.—For the answers of the above questions, See the explanation of Figures 24 and 25, Page 20, of the Relations of Forms.

(See First Table, Fig. 26.)

Q.—Draw a Horizontal Straight line.

Q.—Divide it by one point into two equal parts?

Q.—Draw an Upright Straight line equal to half the horizontal line, above the point of division.

Q.—Draw an Upright Straight line from the point of division below the horizontal line, and equal to half of it.

Q.—How many right angles have you formed here?

Q.—Which is the first right angle?

Q.—What part of the horizontal line forms the horizontal side of the first right angle ?

Q.—What part of the upright line forms the upright side of the first right angle ?

Q.—Which is the second right angle ?

Q.—What part of the horizontal line forms the horizontal side of the second right angle ?

Q.—What part of the upright line forms the upright side of the second right angle.

Q.—Which is the third right angle ?

Q.—What part of the horizontal line forms the horizontal side of the third right angle ?

Q.—What part of the upright line forms the upright side of the third right angle ?

Q.—Which is the fourth right angle ?

Q.—What part of the horizontal line forms the horizontal side of the fourth right angle ?

Q.—What part of the upright line forms the upright side of the fourth right angle ?

Q.—What angle is opposite to the first right angle?

Q.—What angles are adjacent to the first right angle ?

Q.—What angle is opposite to the second right angle ?

Q.—What angles are adjacent to the second right angle?

NOTE.—For the answers to these questions, See the explanation of Fig. 26 Page 21, of the Relations of Forms.

SECT. III.

(See First Table, Fig. 27.)

Q.—Draw a Horizontal Straight line.

Q.—Above the horizontal line, from each extremity of it, draw an Upright line equal to it.

Q.—Join the extremities of these Upright lines by a horizontal line equal to the first.

Q.—What is that figure called?

Q.—How many sides has it?

Q.—How would you distinguish them one from another?

Q.—How many upright sides has it?

Q.—How do you distinguish them one from another?

Q.—How many right angles has this figure?

Q.—How do you distinguish them one from another?

Q.—Where is the first right angle situated?

Q.—What line forms the horizontal side of it?

Q.—What line forms the upright side of it?

Q.—Where is the second right angle situated?

Q.—What line forms the horizontal side of it?

Q.—What line forms the upright side of it?

Q.—Where is the third right angle situated?

Q.—What line forms the horizontal side of it?

Q.—What line forms the upright side of it?

Q.—Where is the fourth right angle situated?

Q.—What line forms the horizontal side of it?

Q.—What line forms the upright side of it?

Q.—At which extremity of the first horizontal line is the first right angle situate?

Q.—At which extremity of the right upright line is the third right angle situated?

Q.—Point out the right lower angle of this square?

Q.—Point out the left lower angle of this square?

Q.—Point out the right upper angle of this square?

Q.—Point out the left upper angle of this square?

NOTE.—For explanation of this Figure see Page 22, of the Relations of Forms

(See First Table, Fig. 28.)

Q.—Draw an Upright Straight line.

Q.—Draw a Horizontal Straight line from the upper extremity of it, towards the right hand, equal to half the Upright line.

Q.—From the lower extremity of the upright line draw another Horizontal line towards the right hand, equal to the upper one.

Q.—Join the right extremity of these two horizontal lines by an upright straight line.

Q.—What is this figure called?

Q.—How many sides has this figure?

Q.—How many horizontal sides has this figure?

Q.—How many upright sides has it?

Q.—How many right angles has it?

Q.—What sides of it are equal to one another?

Q.—How many equal sides must a square have?

Q.—What then is the difference between a rectangle and a square.

NOTE.—The Relations of the Sides and Angles of the Oblong or Rectangle, may then be described the same as of the square; (See the explanation of Figure 28, Page 24, of the Relations of Forms;) nor has it been deemed necessary to give a separate detail for describing and considering the Horizontal Rectangle in Fig. 29.

CHAP. VII.

THE PUPIL having learned in the last Section of the foregoing Chapter, to draw Squares and Rectangles, now proceeds to describe and divide successively, the three lines of Squares, marked 1*st*, 2*d*. 3*d*, on the Table, beginning with those of the first line which are Horizontally divided.

SECT. I.

(See First line of Squares on the First Table of the Relations of Forms.)

Q.—Draw a Horizontal Straight line.

Q.—Divide it by eight points into nine equal parts.

Q.—From each of these eight points of division, and also from the beginning and end of the line, draw an Upright Straight line above the horizontal line, equal to one-ninth of it.

Q.—Join the upper extremities of these upright lines by horizontal lines?

Q.—How many squares have you thus formed?

Q.—Which is the first square? which is the second? which is the third? which is the fourth? which is the fifth? which is the sixth? which is the seventh? which is the eighth? which is the ninth?

Q.—Divide each of the upright sides of the first square by one point into two equal parts.

Q.—Join the points of division by a horizontal straight line.

Q.—Divide each of the upright sides of the second square by two points into three equal parts.

Q.—Join the first point of division on the left upright side to the first point of division on the right upright side, by a horizontal line.

Q.—By how many lines ?

Q.—Into how many horizontal rectangles is the second square divided ?

Q.—By how many lines ?

Q.—Into how many horizontal rectangles is the third square divided ?

Q.—By how many lines ?

Q.—Into how many horizontal rectangles is the fourth square divided ?

Q.—By how many lines ?

Q.—Into how many horizontal rectangles is the fifth square divided ?

Q.—By how many lines ?

Q.—Into how many horizontal rectangles is the sixth square divided ?

Q.—By how many lines ?

Q.—Into how many horizontal rectangles is the seventh square divided ?

Q.—By how many lines ?

Q.—Into how many horizontal rectangles is the eighth square divided ?

Q.—By how many lines ?

Q.—Into how many horizontal rectangles is the ninth square divided?

Q.—By how many lines?

NOTE.—The above should be repeated throughout the second line which consists of Squares divided by Upright lines, and the Pupil having practised the drawing and dividing the squares in the first instance, may (to save time) answer the questions of the following sections on the Table, or by describing only two squares at a time.

SECT. II.

(See Relations of Forms, Page 27.)

Q.—Into how many equal parts is the 1st square of the first line divided?

Q.—What figure is each of the parts?

Q.—What part of the square is each of the parts?

Q—Into how many equal parts is the 2d square of the first line divided?

Q.—What figure is each of the parts?

Q.—What part of the whole square is one of these rectangles?

Q.—What part of the whole square is two of these rectangles?

Q.—What figure does two of these rectangles taken together form?

Q.—How many of these rectangles make up the whole of the second square?

Q.—Into how many equal parts is the 3d square of of the first line divided ?

Q.—What figure is each of the parts ?

Q.—What part of the whole square is one of these rectangles ?

Q.—What part of the whole square is two of these rectangles ?

Q.—What figure does two of these rectangles taken together form ?

Q.—What part of the whole square is three of these rectangles ?

Q.—How many of these rectangles make up the whole of the third square ?

Q.—Into how many equal parts is the 4th square of the first line divided ?

Q.—What figure is each of the parts ?

Q.—What part of the whole square is one of these rectangles ?

Q.—What part of the whole square is two of these rectangles ?

Q.—What figure does two of these rectangles taken together form ?

Q.—What part of the whole square is three of these rectangles.

Q.—What part of the whole square is four of these rectangles?

Q.—How many of these rectangles make up the whole of the fourth square?

Q.—Into how many equal parts is the 5th square of the first line divided?

Q.—What figure is each of the parts?

Q.—What part of the whole square is one of these rectangles?

Q.—What part of the whole square is two of these rectangles?

Q.—What figure does two of these rectangles taken together form?

Q.—What part of the whole square is three of these rectangles?

Q.—What part of the whole square is four of these rectangles?

Q.—What part of the whole square is five of these rectangles?

Q.—How many of these rectangles make up the whole of the fifth square?

Q.—Into how many equal parts is the 6th square of the first line divided?

Q.—What figure is each of the parts ?

Q.—What part of the whole square is one of these rectangles ?

Q.—What part of the whole square is two of these rectangles ?

Q.—What figure does two of these rectangles taken together form ?

Q.—What part of the whole square is three of these rectangles ?

Q.—What part of the whole square is four of these rectangles ?

Q.—What part of the whole square is five of these rectangles ?

Q.—What part of the whole square is six of these rectangles ?

Q.—How many of these rectangles make up the whole of the sixth square ?

Q.—Into how many equal parts is the 7th square of the first line divided ?

Q.—What figure is each of the parts ?

Q.—What part of the whole square is one of these rectangles ?

Q.—What part of the whole square is two of these rectangles ?

Q.—What figure does two of these rectangles taken together form?

Q.—What part of the whole square is three of these rectangles?

Q.—What part of the whole square is four of these rectangles?

Q.—What part of the whole square is five of these rectangles?

Q.—What part of the whole square is six of these rectangles.

Q.—What part of the whole square is seven of these rectangles?

Q.—How many of these rectangles are equal to the whole of the seventh square?

Q.—Into how many equal parts is the 8th square of the first line divided?

Q.—What figure is each of the parts?

Q.—What part of the whole square is one of these rectangles?

Q.—What part of the whole square is two of these rectangles?

Q.—What figure does two of these rectangles taken together form.

Q.—What part of the whole square is three of these rectangles?

Q.—What part of the whole square is four of these rectangles?

Q.—What part of the whole square is five of these rectangles?

Q.—What part of the whole square is six of these rectangles?

Q.—What part of the whole square is seven of these rectangles?

Q.—What part of the whole square is eight of these rectangles?

Q.—How many of these rectangles are equal to the whole of the eighth square?

Into how many equal parts is the 9th square of the first line divided?

Q.—What figure is each of the parts?

Q.—What part of the whole square is one of these rectangles?

Q.—What part of the whole square is two of these rectangles?

Q.—What figure does two of these rectangles taken together form?

Q.—What part of the whole square is three of these rectangles ?

Q.—What part of the whole square is four of these rectangles ?

Q.—What part of the whole square is five of these rectangles ?

Q.—What part of the whole square is six of these rectangles ?

Q.—What part of the whole square is seven of these rectangles ?

Q.—What part of the whole square is eight of these rectangles ?

Q.—What part of the whole square is nine of these rectangles ?

Q.—How many of these rectangles are equal to the whole of the ninth square ?

SECT. III.

(For answers to the following Questions, See Relations of Forms, Page 30.)

Q.—Shew me on the first line of squares where you have a square divided horizontally into two equal rectangles ?

Q.—Shew me where you have a square divided horizontally into three equal rectangles ?

NOTE.—These last two questions should be repeated on commencing the comparison of each new pair of squares.

Q.—Which is greatest, one of the two rectangles into which the first square is divided, or one of the three rectangles into which the second is divided?

Q.—Which is greatest, two of the rectangles of the second square or one of the first?

Q.—Which is greatest, two rectangles of the first or two of the second?

Q.—How many rectangles of the second square are equal to the whole of the first?

Q.—Which is greatest, one of the three rectangles into which the second square of the first line is divided, or one of the four rectangles into which the third square is divided?

Q.—Which is greatest, two of the rectangles of the third square or one of the second?

Q.—Which is greatest, two of the second or two of the third?

Q.—Which is greatest, three of the third or two of the second?

Q.—Which is greatest, three of the second or three of the third?

Q.—How many of the rectangles of the third square are equal to the whole of the second?

Q.—Which is greatest one of the four rectangles into which the third square is divided, or one of the five rectangles into which the 4th square is divided?

Q.—Which is greatest, two rectangles of the fourth square or one of the third?

Q.—Which is greatest, two of the third or two of the fourth?

Q.—Which is greatest, three of the fourth or two of the third?

Q.—Which is greatest, three of the third or three of the fourth?

Q.—Which is greatest, four of the fourth or three of the third?

Q.—Which is greatest, four of the third or four of the fourth?

Q.—How many rectangles of the fourth square are equal to the whole of the third?

Q.—Which is greatest, one of the five rectangles into which the fourth square is divided, or one of the six rectangles into which the fifth square is divided?

Q.—Which is greatest, two of the rectangles of the fifth square or one of the fourth?

Q.—Which is greatest, two of the fourth or two of the fifth?

Q.—Which is greatest, three of the fifth or two of the fourth?

Q.—Which is greatest, three of the fourth or three of the fifth?

Q.—Which is greatest, four of the fifth or three of the fourth ?

Q.—Which is greatest, four of the fourth or four of the fifth ?

Q.—Which is greatest, five of the fifth, or four of the fourth ?

Q.—Which is greatest, five of the fourth or five of the fifth ?

Q.—How many rectangles of the fifth square are equal to the whole of the fourth ?

Q.—Which is greatest, one of the the six rectangles into which the fifth square is divided or one of the seventh rectangles into which the sixth square is divided ?

Q.—Which is the greatest, two of the rectangles of the sixth square or one of the fifth ?

Q.—Which is greatest, two of the fifth or two of the sixth ?

Q.—Which is greatest, three of the sixth or two of the fifth ?

Q.—Which is greatest, three of the fifth or three of the sixth ?

Q.—Which is greatest, four of the sixth or three of the fifth ?

Q.—Which is greatest, four of the fifth or four of the sixth?

Q.—Which is greatest, five of the sixth or four of the fifth?

Q.—Which is greatest, five of the fifth or five of the sixth?

Q.—Which is greatest, six of the sixth or five of the fifth?

Q.—Which is greatest, six of the fifth or six of the sixth?

Q.—How many of the rectangles of the sixth square are equal to the whole of the fifth?

Q.—Which is greatest, one of the seven rectangles into which the sixth square is divided or one of the eight rectangles into which the seventh square is divided?

Q.—Which is greatest, two of the rectangles of the seventh square or one of the sixth?

Q.—Which is greatest, two of the sixth or two of the seventh?

Q.—Which is greatest, three of the seventh or two of the sixth?

Q.—Which is greatest, three of the sixth or three of the seventh?

Q.—Which is greatest, four of the sixth or three of the seventh?

Q.— Which is greatest, five of the seventh or four of the sixth?

Q.—Which is greatest, five of the sixth or five of the seventh?

Q.—Which is greatest, six of the seventh or five of the sixth?

Q.—Which is greatest, six of the sixth or six of the seventh?

Q.—Which is greatest, seven of the seventh or six of the sixth?

Q.—Which is greatest, seven of the sixth or seven of the seventh?

Q.—How many rectangles of the seventh square are equal to the whole of the sixth?

Q.—Which is greatest, one of the eight rectangles into which the seventh square is divided or one of the nine rectangles into which the eighth square is divided?

Q.—Which is greatest, two rectangles of the eighth square or one of the seventh?

Q.—Which is greatest, two of the seventh or two *of the eighth*?

Q.—Which is greatest, three of the eighth or two of the seventh ?

Q.—Which is greatest, three of the seventh or three of the eighth ?

Q.—Which is greatest, four of the eighth or three of the seventh ?

Q.—Which is greatest, four of the seventh or four of the eighth ?

Q.—Which is greatest, five of the eighth or four of the seventh ?

Q.—Which is greatest, five of the seventh or five of the eighth ?

Q.—Which is greatest, six of the eighth or five of the seventh ?

Q.—Which is greatest six of the seventh or six of the eighth ?

Q.—Which is greatest, seven of the eighth or six of the seventh ?

Q.—Which is greatest, seven of the seventh or seven of the eighth ?

Q.—Which is greatest, eight of the eighth or seven of the seventh ?

Q.—Which is greatest, eight of the seventh or eight of the eighth ?

Q.—Which is greatest, nine of the eighth or eight of the seventh ?

Q.—Which is greatest, nine of the seventh or nine of the eighth ?

Q.—How many rectangles of the eighth square are equal to the whole of the seventh ?

Q.—Which is greatest, one of the nine rectangles into which the eighth square is divided or one of the ten rectangles into which the ninth square is divided?

Q.—Which is greatest, two of the rectangles of the ninth square or one of the eighth ?

Q.—Which is greatest two of the eighth or two of the ninth ?

Q.—Which is greatest, three of the ninth or two of the eighth ?

Q.—Which is greatest, three of the eighth or three of the ninth ?

Q.—Which is greatest four of the ninth or three of the eighth ?

Q.—Which is greatest, four of the eighth or four of the ninth ?

Q.—Which is greatest, five of the ninth or four of the eighth ?

Q.—Which is greatest, five of the eighth or five of the ninth?

Q.—Which is greatest, six of the ninth or five of the eighth?

Q.—Which is greatest, six of the eighth or six of the ninth?

Q.—Which is greatest, seven of the ninth or six of the eighth?

Q.—Which is greatest, seven of the eighth or seven of the ninth?

Q.—Which is greatest, eight of the ninth or seven of the eighth?

Q.—Which is greatest, eight of the eighth or eight of the ninth?

Q.—Which is greatest, nine of the ninth or eight of the eighth?

Q.—Which is greatest, nine of the eighth or nine of the ninth?

Q.—How many rectangles of the ninth square are equal to the whole of the eighth?

NOTE.—This mode of comparing the magnitudes of the Rectangles into which the squares are divided, should be repeated on the second line of squares, which is divided by Upright lines, as well as on the first, which is Horizontally divided. The magnitudes should likewise be compared in their decreasing relations, as given in Page 40, of the Relations of Forms; the questions for which it is deemed unnecessary to repeat.

CHAP. VIII.

THE PUPIL having learned on the first lines to divide squares separately into Upright and Horizontal rectangles, and to compare the relations of their several magnitudes, now proceeds on the third line to combine these operations by dividing the same square both Horizontally and Vertically.

SECT. I.

Q.—Draw a Horizontal Straight line.

Q.—Divide it by eight points into nine equal parts.

Q.—From each of the points of division, and also from the beginning and end of the line, draw Upright Straight lines equal to one-ninth of the horizontal line.

Q.—Join the upper extremities of all these upright lines by a horizontal line?

Q.—How many squares have you thus formed?

NOTE.—The nine squares may either be drawn collectively, as above, or perhaps it may be found more convenient to draw only one square at a time, and proceed to divide it and consider the relation of its parts, before we proceed to the remaining squares of the line; this latter method is pursued in the following Exercises:

FIRST SQUARE.

Q.—Draw a Horizontal Straight line.

Q.—Upon it complete a square?

Q.—Divide the horizontal sides of this square by one point into two equal parts?

Q.—Divide the upright sides of this square by one point into two equal parts.

Q.—Draw a Horizontal line joining the points of division of the Upright lines.

Q.—Draw an Upright line joining the points of division of the Horizontal sides.

Q.—Into how many Horizontal Rectangles is this square divided?

Q.—What proportion does the height of each of them bear to its length?

Q.—Into how many Upright Rectangles is this square divided?

Q.—What proportion does the length of each of them bear to its height?

Q.—What part of the whole square is each of these rectangles?

Q.—How many lesser squares does each of these rectangles contain?

Q.—Into how many lesser squares is the whole of the first square divided?

Q.—What part then is each rectangle of the whole square?

SECOND SQUARE.

Q.—Draw a Horizontal Straight line.

Q.—Upon it complete a square.

Q.—Divide the Horizontal sides of this square by two points into three equal parts.

Q.—Divide the Upright sides by two points into three equal parts.

Q.—Join the corresponding points of division of the Upright sides by Horizontal lines.

Q.—Join the corresponding points of division of the Horizontal sides by Upright lines?

Q.—Into how many Horizontal Rectangles is this square divided?

Q.—What proportion does the height of each of them bear to its length?

Q.—Into how many Upright Rectangles is this square divided?

Q.—What proportion does the length of each of them bear to its height?

Q.—What part of the whole square is each of these rectangles?

Q.—How many lesser squares does each of them contain?

Q.—What part of the whole square is two of these rectangles taken together?

Q.—How many lesser squares do two of these contain?

Q.—What proportion does the height of two of them taken horizontally together bear to the length?

Q.—How many lesser squares do the whole of this second square contain?

NOTE.—For the answers to these questions see Pages 46 and 47, of the Relations of Forms.

THIRD SQUARE.

Q.—Draw a Horizontal Straight line.

Q.—Upon it complete a Square.

Q.—Divide the horizontal sides of this square by three points into four equal parts?

Q.—Divide the Upright sides of this square by three points into four equal parts.

Q.—Join the corresponding points of division of the Upright sides by Horizontal lines.

Q.—Join the corresponding points of division of the horizontal sides by Upright lines.

Q.—Into how many horizontal rectangles is this square divided?

Q.—What proportion does the height of each of them bear to its length?

Q.—Into how many upright rectangles is this square divided?

Q.—What proportion does the length of each of them bear to its height?

Q.—What part of the whole square is each of these rectangles?

Q.—How many lesser squares does each of them contain?

Q.—What part of the whole square is two of these rectangles taken together?

Q.—How many lesser squares do two of them contain?

Q.—What proportion does the height of two of them taken horizontally together bear to their length?

Q.—What part of the whole square is three of these rectangles taken horizontally together?

Q.—How many lesser squares do three of them contain?

Q.—What proportion does the height of three of these rectangles taken horizontally together bear to the length?

Q.—How many lesser squares do the whole of the third square contain?

FOURTH SQUARE.

Q.—Draw a Horizontal Straight line?

Q.—Upon it complete a square?

Q.—Divide the horizontal sides of this square by four points into five equal parts?

Q.—Divide the upright sides of this square by four points into five equal parts?

Q.—Join the corresponding points of division of the upright sides by horizontal straight lines.

Q.—Join the corresponding points of division of the horizontal sides by upright straight lines.

Q.—Into how many horizontal rectangles is this square divided?

Q.—What proportion does the height of each of them bear to the length?

Q.—Into how many upright rectangles is this square divided?

Q.—What proportion does the length of each of them bear to its height ?.

Q.—What part of the whole square is each of these rectangles ?

Q.—How many lesser squares do each of them contain ?

Q.—What part of the whole square is two of these rectangles taken horizontally together.

Q.—How many lesser squares do two of them contain ?

Q.—What proportion does the height of two of them taken horizontally together bear to the length ?

Q.—What part of the whole square is three of these rectangles taken horizontally together.

Q.—How many lesser squares do three of them contain ?

Q.—What proportion does the height of three of these rectangles taken horizontally together bear to the length ?

Q.—What part of the whole square is four of these rectangles taken horizontally together ?

Q.—How many lesser squares do four of them contain ?

Q.—What proportion does the height of four of these rectangles taken horizontally together bear to the length?

Q.—How many lesser squares do the whole of this square contain?

FIFTH SQUARE.

Q.—Draw a Horizontal Straight line.

Q.—Upon it complete a square.

Q.—Divide the horizontal sides of this square by five points into six equal parts.

Q.—Divide the upright side of this square by five points into six equal parts.

Q.—Join the corresponding points of division of the upright sides by horizontal lines?

Q.—Join the corresponding points of division of the horizontal sides by upright lines.

Q.—Into how many horizontal rectangles is this square divided?

Q.—What proportion does the height of each of these rectangles bear to its length?

Q.—Into how many upright rectangles is this square divided?

Q.—What proportion does the length of each of them bear to its height?

Q.—What part of the whole square is each of these rectangles?

Q.—How many lesser squares do each of these rectangles contain?

Q.—What part of the whole square is two of these rectangles taken together?

Q.—How many lesser squares do two of them contain?

Q.—What proportion does the height of two of these rectangles taken together horizontally bear to the length?

Q.—What part of the whole square is three of these rectangles taken together?

Q.—How many lesser squares do three of them contain?

Q.—What proportion does the height of three of these rectangles taken together horizontally bear to the length?

Q.—What part of the whole square is four of these rectangles taken together?

Q.—How many lesser squares do four of them contain?

Q.—What proportion does the height of four of these rectangles taken together horizontally bear to the length?

Q.—What part of the whole square is five of these rectangles taken together?

Q.—How many lesser squares do five of them contain?

Q.—What proportion does the height of five of these rectangles taken together horizontally bear to the length?

Q.—How many lesser squares does the whole of this square contain?

SIXTH SQUARE.

Q.—Draw a Horizontal Straight line.

Q.—Upon it complete a square.

Q.—Divide the horizontal sides of this square by six points into seven equal parts.

Q.—Divide the upright sides of this square by six points into seven equal parts?

Q.—Join the corresponding points of division of the upright sides by horizontal lines.

Q.—Join the corresponding points of division of the horizontal sides by upright lines.

Q.—Into how many horizontal rectangles is this square divided ?

Q.—What proportion does the height of each of these rectangles bear to the length ?

Q.—Into how many upright rectangles is this square divided ?

Q.—What proportion does the length of each of these rectangles bear to the height ?

Q.—What part of the whole square is each of these rectangles ?

Q.—How many lesser squares does each of them contain ?

Q.—What part of the whole square is two of these rectangles taken together ?

Q.—How many lesser squares do two of them contain ?

Q.—What proportion does the height of two of them taken together horizontally bear to the length ?

Q.—What part of the whole square is three of these rectangles taken together ?

Q.—How many lesser squares do three of these rectangles contain ?

Q.—What proportion does the height of three of these rectangles taken together horizontally bear to the length?

Q.—What part of the whole square is four of these rectangles taken together?

Q.—How many lesser squares do four of these rectangles contain?

Q.—What proportion does the height of four of these rectangles taken together horizontally bear to the length?

Q.—What part of the whole square is five of these rectangles taken together?

Q.—How many lesser squares do five of these rectangles contain?

Q.—What proportion does the height of five of these rectangles taken together horizontally bear to the length?

Q.—What part of the whole square is six of these rectangles taken together?

Q.—How many lesser squares do six of these rectangles contain?

Q.—What proportion does the height of six of these rectangles taken together horizontally bear to the length?

Q.—How many lesser squares does the whole of this square contain?

SEVENTH SQUARE.

Q.—Draw a Horizontal Straight line.

Q.—Upon it complete a square.

Q.—Divide the horizontal sides of this square by seven points into eight equal parts.

Q.—Divide the upright sides of this square by seven points into eight equal parts.

Q.—Join the corresponding points of division of the upright sides by horizontal lines.

Q.—Join the corresponding points of division of the horizontal sides by upright lines.

Q.—Into how many horizontal rectangles is this square divided?

Q.—What proportion does the height of each of these rectangles bear to the length?

Q.—Into how many upright rectangles is this square divided?

Q.—What proportion does the length of each of these rectangles bear to the height?

Q.—What part of the whole square is each of these rectangles?

Q.—How many lesser squares does each of them contain?

Q.—What part of the whole square is two of these rectangles taken together ?

Q.—How many lesser squares do two of these rectangles contain ?

Q.—What proportion does the height of two of these rectangles taken together horizontally bear to the length ?

Q.—What part of the whole square is three of these rectangles taken together ?

Q.—How many lesser squares do three of these rectangles contain ?

Q.—What proportion does the height of three of these rectangles taken together horizontally bear to the length ?

Q.—What part of the whole square is four of these rectangles taken together ?

Q.—How many lesser squares do four of these rectangles contain ?

Q.—What proportion does the height of four of these rectangles taken together horizontally bear to the length ?

Q.—What part of the whole square is five of these rectangles taken together ?

Q.—How many lesser squares do five of these rectangles contain?

Q.—What proportion does the height of five of these rectangles taken together horizontally bear to the length?

Q.—What part of the whole square is six of these rectangles taken together?

Q.—How many lesser squares do six of these rectangles contain?

Q.—What proportion does the height of six of these rectangles taken together horizontally bear to the length?

Q.—What part of the whole square is seven of these rectangles taken together?

Q.—How many lesser squares do seven of these rectangles contain?

Q.—What proportion does the height of seven of these rectangles taken together horizontally bear to the length?

Q.—How many lesser squares does the whole of this square contain?

EIGHTH SQUARE.

Q.—Draw a Horizontal Straight line.

Q.—Upon it complete a square.

Q.—Divide the horizontal sides of this square by eight points into nine equal parts.

Q.—Divide the upright sides of this square by eight points into nine equal parts.

Q.—Join the corresponding points of division of the upright sides by horizontal lines.

Q.—Join the corresponding points of division of the horizontal sides by upright lines.

Q.—Into how many horizontal rectangles is this square divided?

Q.—What proportion does the height of each of these rectangles bear to the length?

Q.—Into how many upright rectangles is this square divided?

Q.—What proportion does the length of each of these rectangles bear to the height?

Q.—What part of the whole square is each of these rectangles?

Q.—How many lesser squares do each of them contain?

Q.—What part of the whole square is two of these rectangles taken together?

Q.—How many lesser squares do two of these rectangles contain?

Q.—What proportion does the height of two of these rectangles taken together horizontally bear to the length?

Q.—What part of the whole square is three of these rectangles taken together?

Q.—How many lesser squares do three of these rectangles contain?

Q.—What proportion does the height of three of these rectangles taken together horizontally bear to the length?

Q.—What part of the whole square is four of these rectangles taken together?

Q.—How many lesser squares do four of these rectangles contain?

Q.—What proportion does the height of four of these rectangles taken together horizontally bear to the length?

Q.—What part of the whole square is five of these rectangles taken together?

Q.—How many lesser squares do five of these rectangles contain?

Q.—What proportion does the height of five of these rectangles taken together horizontally bear to the length?

Q.—What part of the whole square is six of these rectangles taken together?

Q.—How many lesser squares do six of these rectangles contain?

Q. What proportion does the height of six of these rectangles taken together horizontally bear to the length?

Q.—What part of the whole square is seven of these rectangles taken together?

Q.—What proportion does the height of seven of these rectangles taken together horizontally bear to the length?

Q.—What part of the whole square is eight of these rectangles taken together?

Q.—How many lesser squares do eight of these rectangles contain?

Q.—What proportion does the height of eight of these rectangles taken together horizontally bear to the length?

Q.—How many lesser squares does the whole of this square contain?

NINTH SQUARE.

Q.—Draw a Horizontal Straight line.

Q.—Upon it complete a square.

Q.—Divide the horizontal sides of this square by nine points into ten equal parts.

Q.—Divide the upright sides of this square by nine points into ten equal parts.

Q.—Join the corresponding points of division of the upright sides by horizontal lines.

Q.—Join the corresponding points of division of the horizontal sides by upright lines.

Q.—Into how many horizontal rectangles is this square divided?

Q.—What proportion does the height of each of these rectangles bear to the length?

Q.—Into how many upright rectangles is this square divided?

Q.—What proportion does the length of each of these rectangles bear to the height?

Q.—What part of the whole square is each of these rectangles?

Q.—How many lesser squares do each of these rectangles contain?

Q.—What part of the whole square is two of these rectangles taken together?

Q.—How many lesser squares do two of these rectangles contain ?

Q.—What proportion does the height of two of these rectangles taken together horizontally bear to the length ?

Q.—What part of the whole square is three of these rectangles taken together ?

Q.—How many lesser squares do three of these rectangles contain ?

Q.—What proportion does the height of three of these rectangles taken together horizontally bear to the length ?

Q.—What part of the whole square is four of these rectangles taken together?

Q.—How many lesser squares do four of these rectangles contain ?

Q.—What proportion does the height of four of these rectangles taken together horizontally bear to the length ?

Q.—What part of the whole square is five of these rectangles taken together ?

Q.—How many lesser squares do five of these rectangles contain?

Q.—What proportion does the height of five of

these rectangles taken together horizontally bear to the length ?

Q.—What part of the whole square is six of these rectangles taken together ?

Q.—How many lesser squares do six of these rectangles contain ?

Q.—What proportion does the height of six of these rectangles taken together horizontally bear to the length ?

Q.—What part of the whole square is seven of these rectangles taken together ?

Q.—How many lesser squares do seven of these rectangles contain ?

Q.—What proportion does the height of seven of these rectangles taken together horizontally bear to the length ?

Q.—What part of the whole square is eight of these rectangles taken together ?

Q.—How many lesser squares do eight of these rectangles contain ?

Q.—What proportion does the height of eight of these rectangles taken together horizontally bear to the length ?

Q.—What part of the whole square is nine of these rectangles taken together ?

Q.—How many lesser squares do nine of these rectangles contain ?

Q.—What proportion does the heighth of nine of these rectangles taken together horizontally bear to the length ?

Q.—How many lesser squares does the whole of this square contain ?

CHAP. IX.

HAVING considered in the last Chapter the proportions of the several Rectangles into which a Square can be divided, both by Horizontal and Upright lines, and compared them by means of lesser squares, we now proceed to describe Rectangles bearing a given proportion to a square, or whose height and length have a certain proportion to each other.

SECT. I.

(See line of Rectangles marked A. on the Table.)

Q.—Draw a Horizontal Straight line.

Q.—Below it draw an Upright Straight line from each extremity, equal to half the horizontal line.

Q.—Join the lower extremities of these Upright lines by a Horizontal line?

Q.—What figure is this called?

Q.—What sides of it are equal one to another?

Q.—What proportion do the upright sides of it bear to the horizontal?

SECT. II.

Q.—Draw a Horizontal Straight line.

Below it draw an Upright line from either extremity, equal to one-third of it.

Q.—Join the lower extremities of these upright lines by a horizontal straight line.

Q.—What figure is this called?

Q.—What sides of it are equal?

Q.—What proportion does its height bear to its length?

NOTE.—In like manner should be described a Rectangle whose height is two-thirds of its length; and all the Questions should be asked as above.

SECT. III.

In this Section directions should be given for describing a *Rectangle*, whose height was equal to one-

fourth of its length; and when the questions have been asked concerning its relations, then one whose height is two-fourths of its length; then one whose height is three-fourths of its length.

SECT. IV.

In like manner this Section begins by describing a Rectangle whose height is one-fifth of its length; then one whose height is two-fifths of its length; then, one whose height is three-fifths of its length; then, one whose height is four-fifths of its length; and so on through the remaining rectangles of the line A of the Table, until the Pupil is enabled to describe a rectangle of any proportion, dictated in terms like the following:

Q.—Describe a rectangle placed horizontally, whose height is one-sixth of its length?

Q.—Describe a horizontal rectangle whose height is two-sixths of its length? *And so on.*

Q.—Describe a rectangle placed horizontally whose height is equal to one-seventh of its length?

Q.—Describe a rectangle placed horizontally whose height is equal to three-sevenths of its length?—*And so on.*

Q.— Describe a rectangle placed horizontally whose height is equal to one eighth of its length.

Q.—Describe a rectangle placed horizontally whose height is equal to two-eighths of its length. *And so on.*

Q.—Describe a rectangle placed horizontally whose height is equal to one-ninth of its length.

Q.—Describe a rectangle placed horizontally whose height is equal to two-ninths of its length.—*And so on.*

Q.—Describe a rectangle placed horizontally whose height is equal to one-tenth of its length.

Q.—Describe a rectangle, placed horizontally, whose height is equal to two-tenths of its length.—*And so on.*

NOTE.—The Relations of the different parts of these Rectangles, as given in the questions on the first Section, will be found in Page 56, of the Relations of Forms; and it may be well to state here that, when the height of a rectangle whose sides are upright and horizontal lines, is less than the length, such rectangle is said to be placed *HORIZONTALLY*; and when the height exceeds the length, the rectangle is said to be placed *UPRIGHT*.

We now proceed to the consideration of figures B and C, of the Table; the objects of which are to lay downa rule for observing how much any Slanting line varies from Horizontal, as also in E, arule is laid *down for* ascertaining the slant of those lines whose

inclination approaches nearer to upright. The former, it will appear, are considered as the Diagonals of Rectangles, whose length exceeds their height;—the latter as the Diagonals of Rectangles whose height exceeds their length. We here give a few Questions as examples of their use, which should be carefully extended to every rectangle of the four series, given in the table, and should be dwelt upon, until the Pupil can not only draw a Slanting line of any given inclination, but is also able verbally to express the inclination of any Slanting line presented to his view. It may be well to remark that the line of Rectangles, given in Fig. D of the Table, are intended to be used in the same manner as those at A, in order to facilitate the drawing of any Upright Rectangle before the Diagonals are inserted in them; for which purpose, Questions similar to those given in the foregoing sections of this Chapter, may be used.

Q.—Draw a Rectangle placed horizontally, whose height is equal to one-half of its length.

Q.—Join the right upper Angle to the left lower Angle by a Slanting line.

Q.—What would you say was the inclination of this Slanting line?

Q.—What is a Slanting line drawn thus in a rectangle called?

Q.—In what direction would you say the diagonal of this rectangle was drawn?

NOTE,—For answers to these Questions See Pages 57 and 58 of the Relations of Forms.

The direction of the Diagonal should then be reversed as follows :—

Q.—Draw a Horizontal Rectangle whose height is equal to one-half its length.

Q.—Join the right lower Angle to the left upper Angle by a Slanting line.

Q.—What would you say was the inclination of this Slanting line?

Q.—In what direction would you say this line Slanted?

Q.—Draw an Upright rectangle whose height is equal to twice its length.

Q.—Join its right upper Angle to its left lower Angle by a Slanting line.

Q.—What would you say was the inclination of this line?

Q.—In what direction would you say the diagonal of this rectangle was drawn?

The direction of the Diagonal should be reversed as follows

Q.—Draw an Upright Rectangle whose length is equal to one-half its height.

Q.—Join the right lower Angle to the left upper angle by a Slanting line.

Q.—What would you say was the inclination of this line.

Q.—In what direction would you say the diagonal of this rectangle was formed.

Q.—Draw a Rectangle placed horizontally whose height is equal to one-third of its length.

Q.—Join the right upper Angle to the left lower Angle by a Slanting line.

Q.—What would you say was the inclination of this line?

Q.—What is a Slanting line drawn thus in a rectangle called?

Q.—In what direction would you say the diagonal of this rectangle was drawn?

The direction of the diagonal should be reversed as follows.

Q.—Draw a Horizontal Rectangle whose height is equal to one-third of its lengh.

Q.—Join the right lower Angle to the left upper Angle by a Slanting line.

The direction of the diagonal should be reversed as follows.

Q.—Draw an Upright Rectangle whose height is equal to four times its length.

Q.—Join the right lower Angle to the left upper Angle by a Slanting line.

Q.—What would you say was the inclination of this line?

Q.—In what direction would you say the diagonal of this rectangle was drawn?

In like manner the Diagonals of the remaining Rectangles given on the Table should be described, and afterwards the Diagonals of Rectangles whose heights are two-thirds, or two or three-fourths of its length, &c. &c. as given in describing the Rectangles in the former part of this Chapter; and this Exercise should be continued until the Pupil can, according to this rule, determine, by means of the Rectangle of which it is the Diagonal, the inclination of any line presented to his observation; the practical utility of which is self-evident.

FINIS.

PESTALOZZI'S

RELATIONS OF FORMS.

—00000000—

FIRST EXERCISE.

1ST SECTION.

Horizontal Straight lines.

THE
- 1st
- 2d
- 3d
- 4th
- 5th
- 6th
- 7th
- 8th
- 9th
- 10th

} Horizontal straight line.

See Fig. 1 to 10 1st Table.

Upright straight lines

THE
- 1st
- 2d
- 3d
- 4th
- 5th
- 6th
- 7th
- 8th
- 9th
- 10th

} Upright straight line.

See Fig. 11 to 20. 1st Table.

2D SECTION.

THE first horizontal line is shorter than the second.

The second horizontal line is longer than the first, but shorter than the third.

The third horizontal line is longer than the second but shorter than the fourth.

The fourth horizontal line is longer than the third, but shorter than the fifth.

The fifth horizontal line is longer than the fourth, but shorter than the sixth.

The sixth horizontal line is longer than the fifth, but shorter than the seventh.

The seventh horizontal line is longer than the sixth, but shorter than the eighth.

The eighth horizontal line is longer than the seventh, but shorter than the ninth.

The ninth horizontal line is longer than the eighth, but shorter than the tenth.

The tenth horizontal line is longer than the ninth, and longest of all.

THE first upright line is shorter than the second.

The second upright line is longer than the first, but shorter than the third.

The third upright line is longer than the second, but shorter than the fourth.

The fourth upright line is longer than the third, but shorter than the fifth.

The fifth upright line is longer than the fourth, but shorter than the sixth.

The sixth upright line is longer than the fifth, but shorter than the seventh.

The seventh upright line is longer than the sixth, but shorter than the eighth.

The eighth upright line is longer than the seventh, *but shorter than* the ninth.

The ninth upright line is longer than the eighth, but shorter than the tenth.

The tenth upright line is longer than the ninth, and longest of all.

3D SECTION.

The 1st horizontal line is not divided.

The 2d	is divided by	1	point into	2 equal parts.
3d	-	2	points -	3
4	-	3	-	4
5	-	4	-	5
6	-	5	-	6
7	-	6	-	7
8	-	7	-	8
9	-	8	-	9
10	-	9	-	10

The 1st upright line is not divided.

The 2d	is divided by	1	point into	2 equal parts.
3d	-	2	points -	3
4	-	3	-	4
5	-	4	-	5
6	-	5	-	6
7	-	6	-	7
8	-	7	-	8
9	-	8	-	9
10	-	9	-	10

4TH SECTION.

EACH of the two equal parts of the second horizontal line is half of that line.

From the beginning of the line to the point of division is the first half, and from that point to the end of the line is the second half.

And the whole line contains two halves.

EACH of the three equal parts of the third horizontal line is a third part of that line.

From the beginning of the line to the first point of division is the first third; from the first point to the second is the second third; from the second point to the end of the line is the 3d third.

From the beginning of the line to the second point, there are two thirds of that line, and from the first point to the end of the line there are two thirds.

And the whole line contains three thirds.

EACH of the four equal parts into which the fourth horizontal line is divided, is one fourth part of it.

From the beginning of the line to the first point is the first fourth.

From the 1st point to the 2d point is the 2d fourth.

2d to the 3d point is the 3d fourth.

3d to the end of the line, is the 4th fourth.

From the beginning of the line to the second point, there are two fourths.

From the first point to the 3d there are two fourths

From the second to the end of the line, there are two fourths.

From the beginning of the line to the third point, there are three fourths.

From the 1st point to the end there are three fourths

And the whole line contains four-fourths.

EACH of the five equal parts into which the fifth horizontal line is divided, is one-fifth of it.

From the beginning of the line to the first point of division, is the first fifth.

From the 1st point to the 2d is the 2d fifth.

From the 2d point to the 3d - 3d fifth.

- 3d - 4th - 4th fifth.

- 4th point to end of the line is the 5th fifth

From the beginning of the line to the second poin of division, there are two-fifths.

From the 1st to the 3d there are 2-fifths.

- 2d - 4th - 2-fifths.
- 3d to the end of the line there are 2-fifths.

From the beginning of the line to the third point, there are three-fifths.

From the first point to the fourth, there are three-fifths.

From the second point to the end of the line, there are three-fifths.

From the beginning of the line to the fourth point, there are four-fifths.

From the first point to the end of the line, there are four-fifths.

And the whole line contains five-fifths.

EACH of the six equal parts into which the sixth horizontal line is divided, is one-sixth part of it.

From the beginning of the line to the first point, is the 1st-sixth.

From the 1st point to the 2d, is the 2d sixth.

- 2d - 3d, - 3d
- 3d - 4th, - 4th
- 4th - 5th, - 5th
- 5th point to the end of the line is the 6th sixth.

From the beginning of the line to the 2d poin there are 2-sixths.

From the 1st point to the 3d, there are 2-sixths.

- 2d . 4th . 2
- 3 . 5 . 2
- 4th . 6 . 2
- 5th to the end of the line, 2

From the beginning of the line to the third point there are 3-sixths.

From the 1st point to the 4th there are 3-sixths.

From the 2d point to the 5th there are 3-sixths.

From the 3d point to the end of the line, 3-sixths.

From the beginning of the line to the fourth point there are 4-sixths.

From the 1st to the 5th, are 4-sixths.

From the second to the end of the line are 4-sixths

From the beginning of the line to the 5th point are 5-sixths.

From the 1st point to the end of the line, 5-sixths.

And the whole line contains 6-sixths.

EACH of the seven equal parts into which the seventh horizontal line is divided is 1-7th part of that line.

From the beginning of the line to the first point of division is the 1st seventh.

From the 1st point to the 2d is the 2d seventh.

.	2d	.	3d	.	3d
.	3	.	4th	.	4th
.	4th	.	5	.	5
.	5	.	6	.	6

. 6 . to the end of the line is the 7th seventh.

From the beginning of the line to the second point of division there are 2-sevenths.

From the 1st point to the 3d there are 2-sevenths.

.	2d	.	4th	.	2
.	3	.	5	.	2
.	4th	.	6	.	2

. 5 . to the end of the line there are 2-sevenths.

From the beginning of the line to the 3d point of division there are 3-sevenths.

From the 1st to the 4th point there are 3-sevenths.

From the 2d point to the 5th there are 3-sevenths
. 3d . 6th . . 3-sevenths.
. 4th . to the end of the line there are 3-sevenths.

From the beginning of the line to the fourth point of division there are 4-sevenths.

From the 1st point to the 5th there are 4-sevenths.
. 2d . 6th . 4-sevenths.

From the 3d point to the end of the line there are 4-sevenths.

From the beginning of the line to the fifth point there are 5-sevenths.

From the 1st point to the 6th there are 5-sevenths.

From the 2d point to the end of the line there are 5-sevenths.

From the beginning of the line to the sixth point of division there are 6-sevenths.

From the 1st point to the end of the line there are 6-sevenths.

And the whole line contains 7-sevenths.

EACH of the eight equal parts into which the eighth horizontal line is divided is one-eighth part of that line.

From the beginning of the line to the 1st point of division is the first eighth.

From the 1st point tothe 2d is the 2d eighth
. 2d . 3 . 3d
. 3 . 4th . 4th
. 4th . 5 . 5
. 5 . 6 . 6
. 6 . 7 . 7
. 7th point to the end of the line is the 8th eighth.

From the beginning of the line to the second point of division there are two eighths.

From the 1st point to the 3d there are 2-eighths.
. 2d . 4th . 2
. 3 . 5 . 2
. 4th . 6 . 2
. 5 . 7 . 2
. 6 to the end of the line there are 2-eighths.

From the beginning of the line to the third point of division there are 3-eighths.

From the 1st point to the 4th there are 3-eighths.
. 2d . 5 . 3
. 3 . 6 . 3
. 4th . 7 . 3
. 5th to the end of the line, 3-eighths.

From the beginning of the line to the fourth point of division there are 4-eighths.

From the 1st point to the 5th there are 4-eighths.
. 2d . 6 4
. 3 . 7 . 4
. 4th to the end of the line, 4

From the beginning of the line to the fifth point of division there are 5-eighths.

From the 1st to the 6th there are 5-eighths.
. 2 . 7th . 5
From the 3d to the end of the line, 5-eighths.

From the beginning of the line to the sixth point of division there are 6-eighths.

From the 1st point to the 7th there are 6-eighths
From the 2d to the end of the line, 6-eighths

From the beginning of the line to the seventh point of division there are 7-eighths.

From the 1st point to the end of the line there are 7-eighths.

And the whole line contains 8-eighths.

EACH of the nine equal parts into which the ninth horizontal line is divided is one-ninth part of that line.

From the beginning of the line to the 1st point of division is the first ninth.

From the 1st point to the 2d is the 2d ninth.

.	2d	.	3d		3d
.	3	.	4th	.	4th
.	4th	.	5	.	5
.	5	.	6	.	6
.	6	.	7	.	7
.	7	.	8	.	8

. 8th point to the end of the line is the 9th ninth

From the beginning of the line to the second point of division there are 2-ninths.

From the 1st point to the 3d there are 2-ninths.

.	2d	.	4th	.	2-ninths.
.	3	.	5	.	2
.	4th	.	6	.	2
.	5	.	7	.	2
.	6	.	8	.	2

. 7th to the end of the line, 2

From the beginning of the line to the third point of division there are 3-ninths.

From the 1st point to the 4th there are 3-ninths.

.	2d	.	5	.	3
.	3	.	6	.	3
	4th	.	7	.	3
.	5	.	8	.	3

. 6 to the end of the line, 3

From the beginning of the line to the fourth point of division there are 4-ninths.

From the 1st point to the 5th there are 4-ninths.

.	2	.	6	.	4
.	3	.	7	.	4
.	4th	.	8	.	4

. 5th to the end of the line, 4

From the beginning of the line to the fifth point of division there are 5-ninths.

From the 1st point to the 6th there are 5-ninths.

. 2d . 7 . 5

. 3 . 8 . 5

. 4th to the end of the line, 5

From the beginning of the line to the sixth point of division there are 6-ninths.

From the 1st point to the 7th there are 6-ninths.

. 2d . 8 . 6

. 3d to the end of the line, 6

From the beginning of the line to the to the seventh point of division there are 7-ninths.

From the 1st point to the 8th there are 7-ninths.

From the 2d point to the end of the line there are 7-ninths.

From the beginning of the line to the eighth point of division there are 8-ninths.

From the first point to the end of the line 8-ninths.

And the whole line contains 9-ninths.

EACH of the ten equal parts into which the tenth horizontal line is divided is one-tenth part of that line.

From the beginning of the line to the first point of division, is the first-tenth.

From the 1st point to the 2d, is the 2d tenth.

- 2d - 3 - 3
- 3d - 4th - 4
- 4th - 5 - 5
- 5 - 6 - 6
- 6 - 7 - 7
- 7 - 8 - 8
- 8 - 9 - 9

From the 9th to the end of the line is the 10-tenth.

From the beginning of the line to the second point of division, there are two-tenths.

From the 1st to the 3d point there are 2-tenths.
- 2d - 4th - 2
- 3 - 5 - 2
- 4th - 6 - 2
- 5 - 7 - 2
- 6 - 8 - 2
- 7 - 9 - 2

From the 8th to the end of the line there are 2-tenths.

From the beginning of the line to the third point of division, there are three-tenths.

From the 1st to the 4th point there are 3-tenths.
- 2d - 5th - 3
- 3 - 6 - 3
- 4th - 7 - 3
- 5 - 8 - 3
- 6 - 9 - 3

From the 7th point to the end of the line there are 3-tenths,

From the beginning of the line to the fourth point of division there are four-tenths.

From the 1st to the 5th point there are 4-tenths.
- 2d - 6th - 4
- 3 - 7 - 4
- 4th - 8 - 4
- 5 - 9 - 4
- 6th to end of the line there are 4-tenths.

From the beginning of the line to the fifth point of division there are five-tenths.

From the 1st to the 6th point there are 5-tenths.
- 2 - 7th - 5
- 3 - 8 - 5
- 4th - 9 - 5
- 5 to the end of the line there are 5-tenths

From the beginning of the line to the sixth point of division there are six-tenths.

From the 1st point to the 7th there are 6-tenths.

. 2d . 8 . 6

. 3 . 9 . 6

From the 4th to the end of the line 6

From the beginning of the line to the seventh point of division there are 7-tenths.

From the 1st to the 8th point there are 7-tenths.

. 2d . 9 . 7

. 3 to the end of the line there are 7 tenths

From the beginning of the line to the eighth point of division there are 8-tenths.

From the 1st to the 9th point there are 8-tenths.

. 2d to the end of the line, 8-tenths.

From the beginning of the line to the 9th point of division, there are 9-tenths.

From the first point to the end of the line there are 9-tenths.

And the whole line contains 10-tenths.

NOTE These Exercises should be repeated on the series of upright lines from FIG. 11 to 20, altering the term HORIZONTAL LINE to that of UPRIGHT LINE wherever it occurs.

5TH SECTION.

THE first horizontal line is as great as half the second.

Half the second horizontal line is as great as a 3d part of the third.

The second horizontal line is as great as twice the 3d part of the third.

THE third part of the third horizontal line is as great as the 4th part of the fourth line.

Twice the third part of the third horizontal line is as great as twice the 4th part of the fourth.

The third horizontal line is as great as three times the 4th part of the fourth.

THE fourth part of the fourth line is as great as the 5th part of the fifth.

Twice the 4th part of the fourth horizontal line is equal to twice the 5th part of the fifth line.

Three times the 4th part of the fourth horizontal line. is equal to three times the 5th part of the fifth line

The fourth horizontal line is as great as four times the 5th part of the fifth.

THE 5th part of the fifth horizontal line is as great as the 6th part of the sixth line.

Twice the 5th part of the 5th horizontal line is equal to twice the 6th part of the sixth line.

Three times the 5th part of the fifth horizontal line is equal to three times the 6th part of the sixth line.

Four times the 5th part of the fifth horizontal line is equal to four times the 6th part of the sixth line.

The fifth horizontal line is as great as five times *the 6th part of the* sixth.

THE sixth part of the sixth horizontal line is as great as the seventh part of the seventh.

Twice the 6th part of the sixth horizontal line are equal to twice the 7th part of the seventh line.

Three times the 6th part of the sixth horizontal line are equal to three times the 7th part of the seventh line.

Four times the 6th part of the sixth horizontal line are equal to four times the 7th part of the seventh.

Five times the 6th part of the sixth horizontal line are equal to five times the 7th part of the seventh.

The sixth horizontal line is equal to 6-7ths of the seventh.

THE 7th part of the seventh horizontal line is equal to the 8th part of the eighth.

Twice the 7th part of the seventh horizontal line are equal to twice the 8th part of the eighth

Three times the 7th part of the seventh horizontal line are equal to three times the 8th part of the eighth line.

Four times the 7th part of the seventh horizontal line are equal to four times the 8th part of the eighth line.

Five times the 7th part of the seventh horizontal line are equal to five times the 8th part of the eighth line.

Six times the 7th part of the seventh horizontal line are equal to six times the 8th part of the eighth line.

The seventh horizontal line is equal to seven times the 8th part of the eighth.

THE 8th part of the eighth horizontal line is equal *to the 9th* part of the ninth line.

Twice the 8th part of the eighth horizontal line are *equal to twice* the 9th part of the ninth line.

Three times the 8th part of the eighth horizontal ne are equal to three times the 9th part of the ninth ne.

Four times the 8th part of the eighth horizontal ne are equal to four times the 9th part of the ninth ne.

Five times the 8th part of the eighth horizontal ne are equal to five times the 9th part of the ninth ne.

Six times the 8th part of the eighth horizontal line re equal to six times the ninth part of the ninth line.

Seven times the 8th part of the eighth horizontal ne are equal to seven times the 9th part of the ninth ne.

The eighth horizontal line is equal to 8 times the th part of the ninth line.

THE 9th part of the ninth line is equal to the 10th art of the tenth.

Twice the 9th part of the ninth horizontal line are qual to twice the 10th part of the tenth line

Three times the 9th part of the ninth horizontal ne are equal to three times the 10th part of the tenth ne.

Four times the 9th part of the ninth horizontal ne are equal to four times the 10th part of the tenth ne.

Five times the 9th part of the ninth horizontal ne are equal to five times the 10th part of the tenth ne.

Six times the 9th part of the ninth horizontal line e equal to six times the 10th part of the tenth line.

Seven times the 9th part of the ninth horizontal ne are equal to seven times the 10th part of the tenth ne.

Eight times the 9th part of the ninth horizontal line are equal to eight times the 10th part of the tenth line.

The ninth horizontal line is equal to 9-10ths of the tenth line.

NOTE. The same is to be performed on the Upright lines also.

6TH SECTION.

HALF of the second horizontal line is equal to the whole of the first.

The second horizontal line is twice as great as the first.

The 3d part of the third horizontal line is equal to half of the second.

Twice the 3d part of the third line are equal to the whole of the second.

The whole of the third horizontal line is equal to three times half the second.

THE 4th part of the fourth horizontal line is equal to the 3d part of the third.

Twice the 4th part of the fourth are equal to twice the 3d part of the third.

Three times the 4th part of the fourth are equal to the whole of the third.

The fourth horizontal line is equal to four times the 3d part of the third.

THE 5th part of the fifth line is equal to the 4th part of the fourth.

Twice the 5th part of the fifth horizontal line are equal to twice the 4th part of the fourth line.

Three times the 5th part of the fifth horizontal line are equal to three times the 4th part of the fourth

Four times the 5th part of the fifth horizontal line are equal to the whole of the fourth line.

The fifth line is equal to five times the 4th part of the fourth.

THE 6th part of the sixth line is equal to the 5th part of the fifth line.

Twice the 6th part of the sixth horizontal line are equal to twice the 5th part of the fifth line.

Three times the 6th part of the sixth horizontal line are equal to three times the 5th part of the fifth line.

Four times the 6th part of the sixth horizontal line are equal to four times the 5th part of the fifth line.

Five times the 6th part of the sixth horizontal line are equal to the whole of the fifth line.

The sixth horizontal line is equal to six times the 5th part of the fifth.

THE 7th part of the seventh line is equal to the 6th part of the sixth.

Twice the 7th part of the seventh horizontal line are equal to twice the 6th part of the sixth line.

Three times the 7th part of the seventh horizontal line are equal to three times the 6th part of the sixth line.

Four times the 7th part of the 7th horizontal line are equal to the four times the 6th part of the sixth line.

Five times the 7th part of the seventh horizontal line are equal to five times the 6th part of the sixth line.

Six times the 7th part of the seventh horizontal line are equal to the whole of the sixth line.

The seventh horizontal line is equal to seven times the sixth part of the sixth.

THE 8th part of the eighth line is equal to the 7th part of the seventh line.

Twice the 8th part of the eighth horizontal line are equal to twice the 7th part of the seventh line.

Three times the 8th part of the eighth horizontal line are equal to three times the 7th part of the seventh line.

Four times the 8th part of the eighth horizontal line are equal to four times the 7th part of the seventh line.

Five times the 8th part of the eighth horizontal line are equal to five times the 7th part of the seventh line.

Six times the 8th part of the eighth horizontal line are equal to six times the 7th part of the seventh line.

Seven times the 8th part of the eighth horizontal line are equal to the whole of the seventh line.

The eighth horizontal line is equal to eight times the 7th part of the seventh line.

THE 9th part of the ninth line is equal to the 8th part of the eighth.

Twice the 9th part of the ninth horizontal line is equal to twice the 8th part of the eighth line.

Three times the 9th part of the ninth horizontal line are equal to three times the 8th part of the eighth line.

Four times the 9th part of the ninth horizontal line are equal to four times the 8th part of the eighth line.

Five times the 9th part of the ninth horizontal line are equal to five times the 8th part of the eighth line.

Six times the 9th part of the ninth horizontal line *equal to* six times the 8th part of the eighth

Seven times the 9th part of the ninth horizontal line are equal to seven times the 8th part of the eighth line.

Eight times the 9th part of the ninth horizontal line are equal to the whole of the eighth.

The ninth horizontal line is equal to nine times the 8th part of the eighth.

THE tenth part of the tenth line is equal to the 9th part of the ninth line.

Twice the 10th part of the tenth horizontal line are equal to twice the 9th part of the ninth.

Three times the 10th part of the tenth horizontal line are equal to three times the 9th part of the ninth.

Four times the 10th part of the tenth horizontal line are equal to four times the 9th part of the ninth.

Five times the 10th part of the tenth horizontal line are equal to five times the 9th part of the ninth.

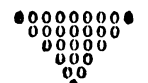
Six times the 10th part of the tenth horizontal line are equal to six times the 9th part of the ninth.

Seven times the 10th part of the tenth horizontal line are equal to seven times the 9th part of the ninth.

Eight times the 10th part of the ninth horizontal line are equal to eight times the 9th part of the ninth.

Nine times the 10th part of the tenth horizontal line are equal to the whole of the ninth line.

The tenth horizontal line is equal to ten times the 9th part of the ninth.



SECOND EXERCISE.

FIG. 21

HORIZONTAL PARALLEL LINES.

THE upper horizontal line is parallel to the lower, and the lower hor. line is parallel to the upper, because both are equally distant one from another in every part.

FIG. 22.

UPRIGHT PARALLEL LINES.

The first upright line is parallel to the second, and the second upright line is parallel to the first, because both are equally distant in every part.

FIG. 23.

A RIGHT ANGLE.

This Right Angle is composed of one upright line and one horizontal line.

The horizontal line is the horizontal side of the angle.

The upright line is the upright side of the angle.

The point where these two lines meet is called the Point, or Vertex of the angle.

FIG. 24.

TWO RIGHT ANGLES.

These two Right Angles are formed by one hor line and one vertical.

The first half of the horizontal line is the hor. side of the first right angle.

The second half of the hor. line is the hor. side of *the second right* angle.

The upright line is the upright side of the first right angle, and also of the second right angle for which reason they are called Adjacent Angles.

The point where the upright line meets the hor. line is called the vertex, or point of both these angles.

FIG. 25.

The Exercises of this figure are the same as the preceding one.

FIG. 26.

FOUR RIGHT ANGLES.

(a) The 1st right angle. (b) The 2d right angle.
(c) The 3d right angle. (d) The 4th right angle.

The first half of the hor. line and the upper half of the upright line, are the sides of the first right angle. (a)

The upper half of the upright line and the second half of the hor. line, are the sides of the second right angle. (b)

The first half of the hor. and the lower half of the upright line, are the sides of the third right angle. (c)

The second half of the hor. and the lower half of the upright line, are sides of the fourth right angle. (d)

The upper half of the upright line is the upright side of the first right angle, and also of the second right angle; these two angles are, therefore, adjacent to each other.

The lower half of the upright line is the upright side of the third right angle, and also of the fourth right angle; these two angles are, therefore, adjacent to each other.

The 1st half of the horizontal line is a hor. side of the first right angle, and also of the third right angle; these angles are, therefore, adjacent one to the other.

The second half of the horizontal line is a hor. side of the second and fourth right angles; these angles are, therefore, adjacent one to another.

The point where the upright line cuts the hor. line is called the vertex, or point of the four right angles.

The horizontal side of the first right angle, and of the fourth right angle, are in the same straight line; and the upright side of the first right angle is also in the same straight line with the upright side of the 4th right angle; these two angles, in such case, are called opposite angles one to another.

The horizontal side of the second right angle is in the same straight line with the hor. side of the third right angle, and the upright side of the second right angle is also in the same straight line with the upright side of the third right angle; these two angles are, therefore, said to be opposite one to another.

FIG. 27.

OF A SQUARE.

This four sided figure is formed of four lines.

Each of these four lines is a side of the four sided figure.

Two of these four lines are horizontal.

Two of the four lines are upright.

The horizontal lines form the horizontal sides of the square.

The upright lines from the upright sides of the square.

These four lines united, form a four sided figure, having four right angles.

The upper hor. line and the upright line next the left hand, form the first right angle.

The upper hor. line and the upright line next the *right hand,* form the second right angle.

he upright line next the left hand and the lower line form the third right angle.

The upright line next the right hand and the lower, hor. line, form the fourth right angle.

The upper extremities of both upright lines meet the extremities of the upper hor. line, forming the upper angles of the square.

The lower extremities of the two upright lines of this square meet the extremities of the lower hor. line and form with them the lower angles of the square.

The point where the upper hor. line meets the left upright line forms the vertex of the left upper angle of the square.

The point where the upper hor. line meets the right upright line forms the vertex of the right upper angle of the square.

The point where the left upright line meets the lower hor. line, forms the vertex of the left lower angle of the square.

The point where the right upright line meets the lower hor, line, forms the vertex of the right lower angle of the square.

The left upper point of junction of this square is the vertex of the first right angle.

The right upper point of junction of this square is the vertex of the second right angle.

The left lower point of junction of this square is thevertex of the third right angle.

The right lower point of junction of this square is the vertex of the fourth right angle.

And because the four angles of this four sided figure are all four right angles, it is called a right angled four sided figure. And when, moreover, the four sides of this figure are all equal, it is called a square.

That is the upper hor. side is equal to the left *upright.*

The left upright side is equal to the low
The lower hor. is equal to the right up
The right upright is equal to the uppe
And for this reason the four-sided figure
square.

FIG. 28TH.

AN OBLONG, OR RECTANGLE.

This four-sided figure is formed with four straight lines.

Each of these four sides is a side of the four-sided figure.

Of these four lines, two are horizontal;

The other two are upright.

The horizontal lines form the horizontal sides of the figure.

The upright lines form the upright sides of the figure.

These four lines meet and form a four-sided figure, having four right angles.

The upper hor. line and the left upright line form the first right angle.

The upper hor. line and the right upright line form the second right angle.

The left upright line and the lower hor. line form the 3d right angle.

The right upright line and the lower hor. line form the 4th right angle.

The upper extremities of the two upright lines meeting the extremities of the upper hor. line form the two upper angles of the figure.

The lower extremities of the two upright lines meeting *the extremities* of the lower hor. line form the two *lower angles* of the figure.

The point where the upper horizontal line meets the left upright line, is the vertex of the left upper angle.

The point where the right upright line meets the upper horizontal line is the vertex of the right upper angle.

The point where the left upright line meets the left horizontal line is the vertex of the left lower angle of the square.

The point where the right upright line meets the lower horizontal line is the vertex of the right lower angle.

The left upper point of junction is the vertex of the first angle.

The right upper point of junction is the vertex of the second angle.

The left lower point of junction is the vertex of the third right angle.

The right lower point of junction is the vertex of the fourth right angle.

The horizontal sides of this four-sided figure are shorter than the upright ones.

The upper horizontal side of this four-sided figure is shorter than the left upright side adjacent to it.

The upper horizontal side of this four-sided figure is shorter than the right upright side adjacent to it.

The left upright side is greater than the lower hor. side adjacent to it.

The right upright side is greater than the lower horizontal side adjacent to it.

The upper horizontal side is equal to the lower hor. side.

The left upright side is equal to the right upright side.

And because this four-sided figure is composed of two horizontal sides, equal to one another, and, also, of two upright sides, respectively equal to one another, and all forming right angles, this figure is called an Oblong or Rectangle.

FIG. 29TH.

THE explanation of this figure is the same as the foregoing, only with the difference of changing the words Upright & Horizontal, according to the difference of their position. And it will not be difficult to lead on the mind of the child to conceive that the figure described in No. 28, may be moved so as to lie in the position of No. 29; by which its actual quantities will not be altered, but only the position of its sides will have changed places.

THIRD EXERCISE.

THE first line contains a series of Squares divided by horizontal lines.

The 1st square is divided by 1 hor. line into 2 equal parts.

The 2d	-	-	2	-	- 3
3d	-	-	3	-	- 4th
4th	-	-	4th	-	- 5
5	-	-	5	-	- 6
6	-	-	6	-	- 7
7	-	-	7	-	- 8
8	-	-	8	-	- 9
9	-	-	9	-	- 10

2D LINE.

THE second line contains a series of squares divided *by upright lines.*

The 1st square is divided by 1 upright line into 2 equal parts.

The 2d square is divided by 2 upright lines into 3 equal parts.

3d	-	-	3	-		4
4th	-	-	4	-	-	5
5	-	-	5	-	-	6
6	-	-	6	-	-	7
7	-	-	7	-	-	8
8	-	-	8	-	-	9
9	-	-	9	-	-	10

Upon the 1st LINE—Each of the two equal parts of the first square is a rectangle or oblong, and is half of the square.

EACH of the three equal parts of the second square is a rectangle or oblong, and is one-third of the whole square.

Two of these equal parts when taken together are also a rectangle, or oblong, equal to two-thirds of the whole square.

EACH of the four equal parts of the 3d square is a rectangle or oblong, equal to one-fourth of the whole square.

Two of these equal parts taken together form a rectangle or oblong, equal to 2-4ths of the whole square.

Three of these equal parts taken together are a rectangle or oblong, equal to three-fourths of the whole square.

NOTE. The same should be practised all through upon the second line which consists of squares similarly divided by upright lines; it is therefore accounted unnecessary to repeat the details.

EACH of the five equal parts of the fourth square is a rectangle or oblong, equal to one-fifth of the whole square,

Two of these equal parts taken together are also a rectangle or oblong, equal to two-fifths of the whole square.

Three of these equal parts taken together are a rectangle or oblong, equal to 3-5ths of the whole sq.

Four of these equal parts taken together are a rectangle or oblong, equal to four-fifths of the whole sq.

EACH of the six equal parts of the fifth square is a rectangle or oblong, equal to 1-6th of the whole sq.

Two of these equal parts taken together are a rectangle or oblong, equal to 2-6ths of the whole square.

Three of these equal parts taken together are a rectangle or oblong, equal to 3-6ths of the whole square.

Four of these equal parts taken together are a rectangle or oblong, equal to 4-6ths of the whole square.

Five of these equal parts taken together are a rectangle or oblong, equal to 5-6ths of the whole square.

EACH of the seven equal parts of the sixth square is a rectangle or oblong, equal to 1-7th of the whole square.

Two of these equal parts taken together are a rectangle or oblong, equal to 2-7ths of the whole square.

Three of these equal parts taken together are a rectangle or oblong, equal to 3-7ths of the whole square.

Four of these equal parts taken together are a rectangle or oblong, equal to 4-7ths. of the whole square.

Five of these equal parts taken together are a rectangle or oblong, equal to 5-7ths. of the whole square.

Six of these equal parts taken together are a *rectangle or oblong,* equal to 6-7ths. of the whole square

EACH of the eight equal parts of the seventh square is a rectangle or oblong, equal to 1-8th of the square.

Two of these equal parts taken together are a rectangle or oblong, equal to 2-8ths. of the whole square.

Three of these equal parts taken together are a rectangle or oblong, equal to 3-8ths. of the whole square.

Four of these equal parts taken together are a rectangle or oblong, equal to 4-8ths of the whole square.

Five of these equal parts taken together are a rectangle or oblong, equal to 5-8ths of the whole square.

Six of these equal parts taken together are a rectangle or oblong, equal to 6-8ths. of the whole square.

Seven of these equal parts taken together are a rectangle or oblong, equal to 7-8ths. of the whole square.

EACH of the nine equal parts of the eighth square is a rectangle or oblong, equal to 1-9th of the whole square.

Two of these equal parts taken together are a rectangle or oblong, equal to 2-9ths of the whole square.

Three of these equal parts taken together are a rectangle or oblong, equal to 3-9ths of the whole square.

Four of these equal parts taken together are a rectangle or oblong, equal to 4-9ths of the whole square.

Five of these equal parts taken together are a rectangle or oblong, equal to 5-9ths of the whole square.

Six of these equal parts taken together are a rectangle or oblong, equal to 6-9ths of the whole square.

Seven of these equal parts taken together are a rectangle or oblong, equal to 7-9ths of the whole square.

Eight of these equal parts taken together are a rectangle or oblong, equal to 8-9ths of the whole square.

EACH of the ten equal parts of the ninth square is a rectangle or oblong, and is 1-10th of the whole square.

Two of these equal parts taken together are a rectangle or oblong, equal to 2-10ths of the whole square.

Three of these equal parts taken together are a rectangle or oblong, equal to 3-10ths of the whole square.

Four of these equal parts taken together are a rectangle or oblong, equal to 4-10ths of the whole square.

Five of these equal parts taken together are a rectangle or oblong, equal to 5-10ths of the whole square.

Six of these equal parts taken together are a rectangle or oblong, equal to 6-10ths of the whole square.

Seven of these equal parts taken together are a rectangle or oblong, equal to 7-10ths of the whole square.

Eight of these equal parts taken together are a rectangle or oblong, equal to 8-10ths of the whole square.

Nine of these equal parts taken together are a rectangle or oblong, equal to 9-10ths of the whole square.

ONE of the two equal parts of the first square, or a rectangle, equal to half the square, is greater than one of the three equal parts of the second square, or than a rectangle equal to a third part of the second square.

Two of the three equal parts of the second square, or a rectangle equal to two-thirds of the second square, is greater than one of the two equal parts of the first square, or than a rectangle equal to half of the first square.

The whole of the first square is greater than *two-thirds of* the second square; or than a rectangle equal *to two-third* parts of the square.

The second square is equal to the first.

ONE of the three equal parts of the second square, r a rectangle equal to one-third of the square, is reater than one of the four equal parts of the third quare, or a rectangle equal to a fourth part of the hird square.

Two of the four equal parts of the third square, or a ectangle equal to two fourth of the square, are greater han one of the three equal parts of the second square, r a rectangle equal to one-third of the square.

Two of the three equal parts of the second square, r a rectangle equal to two-thirds of the square, are reater than two of the four equal parts of the third quare, or a rectangle equal to two-fourths of the quare.

Three of the four equal parts of the third square, a rectangle equal to three-fourths of the square, e greater than two of the three equal parts of the cond square, or a rectangle equal to two-thirds of e square.

The whole of the second square is greater than three urths of the third square, or than a rectangle equal three-fourth parts of a square.

The third square is equal to the second.

ONE of the four equal parts of the third square, or rectangle equal to one-fourth of the square, is greater an one of the five equal parts of the fourth square, a rectangle equal to one-fifth part of the fourth uare.

Two of the five equal parts of the fourth squre, or rectangle, equal to two-fifths of the square, are eater than one of the four equal parts of the third uare, *or* a rectangle equal to one-fourth of the uare.

EACH of the five equal parts of the fourth square is a rectangle or oblong, equal to one-fifth of the whole square,

Two of these equal parts taken together are also a rectangle or oblong, equal to two-fifths of the whole square.

Three of these equal parts taken together are a rectangle or oblong, equal to 3-5ths of the whole sq.

Four of these equal parts taken together are a rectangle or oblong, equal to four-fifths of the whole sq.

EACH of the six equal parts of the fifth square is a rectangle or oblong, equal to 1-6th of the whole sq.

Two of these equal parts taken together are a rectangle or oblong, equal to 2-6ths of the whole square.

Three of these equal parts taken together are a rectangle or oblong, equal to 3-6ths of the whole square.

Four of these equal parts taken together are a rectangle or oblong, equal to 4-6ths of the whole square.

Five of these equal parts taken together are a rectangle or oblong, equal to 5-6ths of the whole square.

EACH of the seven equal parts of the sixth square is a rectangle or oblong, equal to 1-7th of the whole square.

Two of these equal parts taken together are a rectangle or oblong, equal to 2-7ths of the whole square.

Three of these equal parts taken together are a rectangle or oblong, equal to 3-7ths of the whole square.

Four of these equal parts taken together are a rectangle or oblong, equal to 4-7ths. of the whole square.

Five of these equal parts taken together are a rectangle or oblong, equal to 5-7ths. of the whole square.

Six of these equal parts taken together are a rectangle *or oblong*, equal to 6-7ths. of the whole square

EACH of the eight equal parts of the seventh square is a rectangle or oblong, equal to 1-8th of the square.

Two of these equal parts taken together are a rectangle or oblong, equal to 2-8ths. of the whole square.

Three of these equal parts taken together are a rectangle or oblong, equal to 3-8ths. of the whole square.

Four of these equal parts taken together are a rectangle or oblong, equal to 4-8ths of the whole square.

Five of these equal parts taken together are a rectangle or oblong, equal to 5-8ths of the whole square.

Six of these equal parts taken together are a rectangle or oblong, equal to 6-8ths. of the whole square.

Seven of these equal parts taken together are a rectangle or oblong, equal to 7-8ths. of the whole square.

EACH of the nine equal parts of the eighth square is a rectangle or oblong, equal to 1-9th of the whole square.

Two of these equal parts taken together are a rectangle or oblong, equal to 2-9ths of the whole square.

Three of these equal parts taken together are a rectangle or oblong, equal to 3-9ths of the whole square.

Four of these equal parts taken together are a rectangle or oblong, equal to 4-9ths of the whole square.

Five of these equal parts taken together are a rectangle or oblong, equal to 5-9ths of the whole square.

Six of these equal parts taken together are a rectangle or oblong, equal to 6-9ths of the whole square.

Seven of these equal parts taken together are a rectangle or oblong, equal to 7-9ths of the whole square.

Eight of these equal parts taken together are a rectangle or oblong, equal to 8-9ths of the whole square.

EACH of the ten equal parts of the ninth square is a rectangle or oblong, and is 1-10th of the whole square.

Two of these equal parts taken together are a rectangle or oblong, equal to 2-10ths of the whole square.

Three of these equal parts taken together are a rectangle or oblong, equal to 3-10ths of the whole square.

Four of these equal parts taken together are a rectangle or oblong, equal to 4-10ths of the whole square.

Five of these equal parts taken together are a rectangle or oblong, equal to 5-10ths of the whole square.

Six of these equal parts taken together are a rectangle or oblong, equal to 6-10ths of the whole square.

Seven of these equal parts taken together are a rectangle or oblong, equal to 7-10ths of the whole square.

Eight of these equal parts taken together are a rectangle or oblong, equal to 8-10ths of the whole square.

Nine of these equal parts taken together are a rectangle or oblong, equal to 9-10ths of the whole square.

ONE of the two equal parts of the first square, or a rectangle, equal to half the square, is greater than one of the three equal parts of the second square, or than a rectangle equal to a third part of the second square.

Two of the three equal parts of the second square, or a rectangle equal to two-thirds of the second square, is greater than one of the two equal parts of the first square, or than a rectangle equal to half of the first square.

The whole of the first square is greater than two-*thirds of* the second square; or than a rectangle equal *to two-third* parts of the square.

The second square is equal to the first.

ONE of the three equal parts of the second square, or a rectangle equal to one-third of the square, is greater than one of the four equal parts of the third square, or a rectangle equal to a fourth part of the third square.

Two of the four equal parts of the third square, or a rectangle equal to two fourth of the square, are greater than one of the three equal parts of the second square, or a rectangle equal to one-third of the square.

Two of the three equal parts of the second square, or a rectangle equal to two-thirds of the square, are greater than two of the four equal parts of the third square, or a rectangle equal to two-fourths of the square.

Three of the four equal parts of the third square, or a rectangle equal to three-fourths of the square, are greater than two of the three equal parts of the second square, or a rectangle equal to two-thirds of the square.

The whole of the second square is greater than three fourths of the third square, or than a rectangle equal to three-fourth parts of a square.

The third square is equal to the second.

ONE of the four equal parts of the third square, or a rectangle equal to one-fourth of the square, is greater than one of the five equal parts of the fourth square, or a rectangle equal to one-fifth part of the fourth square.

Two of the five equal parts of the fourth squre, or a rectangle, equal to two-fifths of the square, are greater than one of the four equal parts of the third square, or a rectangle equal to one-fourth of the *square.*

Two of the four equal parts of the 3d square, or a rectangle, equal to 2-4ths of the square, are greater than two of the five equal parts of the 4th square, or a rectangle equal to 2-5ths of the square.

Three of the five equal parts of the 4th square, or a rectangle, equal to 3-5ths of the square, are greater than two of the four equal parts of the 3d square, or a rectangle, equal to 2-4ths of the square.

Three of the four equal parts of the 3d square, or a rectangle, equal to 3-4ths of the square, are greater than three of the five equal parts of the 4th square, or a rectangle, equal to 3-5ths of the square.

Four of the five equal parts of the 4th square, or a rectangle, equal to 4-5ths of the square, are greater than three of the four equal parts of the 3d square, or a rectangle, equal to 3-4ths of the square.

The whole of the 3d square is greater than 4-5ths of the 4th square, or than a rectangle, equal to 4-5ths of the square.

The 4th square is equal to the 3d.

ONE of the five equal parts of the 4th square, or a rectangle, equal to 1-5th of the square, is greater than one of the six equal parts of the 5th square, or a rectangle equal to 1-6th of the square.

Two of the six equal parts of the 5th square, or a rectangle equal to 2-6ths of the square, are greater than one of the five equal parts of the 4th square, or a rectangle, equal to 1-5th of the square.

Two of the five equal parts of the 4th square, or a rectangle, equal to 2-5ths of the square, are greater than two of the six equal parts of the 5th square, or *a rectangle* equal to 2-6ths of the square.

Three of the six parts of the 5th sq. or a rectangle equal to 3-6th of the sq. are greater than two of the five parts of the 4th sq. or a rectangle equal to 2..5th of the sq.

Three of the five equal parts of the fourth square, or a rectangle equal to 3-5ths of the square are greater than three of the six equal parts of the 5th square, or a rectangle equal to 3-6ths of the square.

Four of the six equal parts of the 5th square, or a rectangle, equal to 4-6ths of the square, are greater than three of the five equal parts of the 4th square, or a rectangle, equal to 3-5ths of the square.

Four of the five equal parts of the 4th square, or a rectangle equal to 4-5ths of the square, are greater than four of the six equal parts of the 5th square, or a rectangle equal to 4-6ths of the square.

Five of the six equal parts of the 5th square, or a rectangle equal to 5-6ths of the square, are greater than four of the five equal parts of the 4th square, or a rectangle equal to 4-5ths of the square.

The whole of the 4th square is greater than 5-6ths of the 5th square, or a rectangle equal to 5-6ths of the square.

The 5th square is equal to the 4th.

ONE of the six equal parts of the 5th square, or a rectangle, equal to 1-6th of the square, is greater than one of the seven equal parts of the 6th square, or a rectangle, equal to 1-7th of the square.

Two of the seven equal parts of the 6th square, or a rectangle equal to 2-7ths of the square, are greater than one of the six equal parts of the 5th square, or a rectangle equal to 1-6th of the square.

four of the seven parts of the 6th square, or a rectangle equal to 4-7ths of the square.

Five of the seven parts of the 6th square or a rectangle equal to 5-7ths of the square, are greater than five of the eight parts of the 7th square, or a rectangle equal to 5-8ths of the square.

Six of the eight parts of the 7th square, or a rectangle equal to 6-8ths of the spuare, are greater than five of the seven parts of the 6th square, or a rectangle equal to 5-7ths of the square.

Six of the seven parts of the 6th square, or a rectangle equal to 6-7ths of the square, are greater than six of the eight parts of the 7th square, or a rectangle equal to 6-8ths of the square.

The whole of the 6th square is greater than 7-8ths of the 7th square, or than a rectangle equal to 7-8th part of the square.

The 7th square is equal to the 6th.

ONE of the eight equal parts of the 7th square, or a rectangle equal to 1-8th of the square, is greater than one of the nine equal parts of the 8th square, or a reetangle equal to 1-9th of the 8th square.

Two of the nine parts of the 8th square or a rect. angle equal to 2-9ths of the square, are greater than one of the eight parts of the 7th square, or a rectangle equal to 1-8th of the square.

Two of the eight parts of the 7th square, or a rectangle equal to 2-8ths of the sq., are greater than two of the nine parts of the 8th sq., or a rectangle equal to 2-9ths of the sq.

Three of the nine parts of the 8th sq., or a rectangle equal to 3-9ths of the sq., are greater than two of the *eight parts* of the 7th sq, or a rectangle equal to 2-8ths *of the square*

Three of the eight parts of the 7th square, or a rectangle equal to 3-8ths of the 7th square, are greater than three of the nine equal parts of the 8th square, or a rectangle equal to 3-9ths of the square.

Four of the nine parts of the 8th square, or a rectangle equal to 4-9ths of the 8th square, are greater than three of the eight parts of the 7th square, or a rectangle equal to 3-8ths of the 7th square.

Four of the eight parts of the 7th square, or a rectangle equal to 4-8ths of the 7th square, are greater than four of the nine parts of the 8th square, or a rectangle equal to 4-9ths of the 8th square.

Five of the nine parts of the 8th square, or a rectangle equal to 5 9ths of the 8th square, are greater than four of the eight parts of the 7th square, or a rectangle equal to 4-8ths of the 7th square.

Five of the eight parts of the 7th square, or a rectangle equal to 5-8ths of the 7th square, are greater than five of the nine parts of the 8th square, or than a rectangle equal to 5-9ths of the 8th square.

Six of the nine parts of the 8th square, or a rectangle equal to 6-9ths of the 8th square, are greater than five of the eight parts of the 7th square, or than a rectangle equal to 5-8ths of the 7th square.

Six of the eight parts of the 7th square, or a rectangle equal to 6-8ths of the 7th square, are greater than six of the nine equal parts of the 8th square, or a rectangle equal to 6-9ths of the 8th square.

Seven of the nine parts of the 8th square, or a rectangle equal to 7-9ths of the 8th square, are greater than six of the eight parts of the 7th square, or a rectangle equal to 6-8ths of the 7th square.

Seven of the eight equal parts of the 7th square, or a rectangle equal to 7-8ths of the 7th square, are greater than seven of the nine parts of the 8th square, or than a rectangle equal to 7-9ths of the 8th square.

Eight of the nine parts of the 8th square, or a rectangle equal to 8-9ths of the 8th square, are greater than seven of the eight parts of the 7th square, or a rectangle equal to 7-8ths of the 7th square.

The whole of the 7th square is greater than 8-9ths of the 8th square, or than a rectangle equal to 8-9ths of the 8th square.

The 8th square is equal to the 7th.

ONE of the nine equal parts of the 8th square, or a rectangle equal to 1-9th of the 8th square, is greater than one of the ten equal parts of the 9th square, or a rectangle equal to 1-10th of the 9th square.

Two of the ten parts of the 9th square, or a rectangle equal to 2-10ths of the 9th square, are greater than one of the nine parts of the 8th square, or than a rectangle equal to 1-9th of the 8th square.

Two of the nine parts of the 8th square, or a rectangle equal to 2-9ths of the 8th square, are greater than two of the ten parts of the 9th square, or a rectangle equal to 2-10ths of the 9th square.

Three of the ten parts of the 9th square, or a rectangle equal to 3-10ths of the 9th square, are greater than two of the nine parts of the 8th square, or a rectangle equal to 2-9ths of the 8th square.

Three of the nine parts of the 8th square, or a rectangle equal to 3-9ths of the 8th square, are greater than three of the ten parts of the 9th square, or a rectangle equal to 3-10ths of the 9th square.

Four of the ten parts of the 9th square, or a rect-

angle equal to 4-10ths of the 9th square, are greater than three of the nine parts of the 8th square, or a rectangle equal to 3-9ths of the 8th square.

Four of the nine parts of the 8th square, or a rectangle equal to 4-9ths of the 8th square, are greater than four of the ten parts of the 9th square, or a rectangle equal to 4-10ths of the 9th square.

Five of the ten parts of the 9th square, or a rectangle equal to 5-10ths of the 9th square, are greater than four of the nine parts of the 8th square, or than a rectangle equal to 4-9ths of the 8th square.

Five of the nine parts of the 8th square, or a rectangle equal to 5-9ths of the 8th square, are greater than five of the ten parts of the 9th square, or a rectangle equal to 5-10ths of the 9th square.

Six of the ten parts of the 9th square, or a rectangle equal to 6-10ths of the 9th square, are greater than five of the nine parts of the 8th square, or than a rectangle equal to 5-9ths of the 8th square.

Six of the nine parts of the 8th square, or a rectangle equal to 6-9ths of the 8th square, are greater than six of the ten parts of the 9th square, or a rectangle equal to 6-10ths of the 9th square.

Seven of the ten parts of the 9th square, or a rectangle equal to 7-10ths of the 9th square, are greater than six of the nine parts of the 8th square, or a rectangle equal to 6-9ths of the 8th square.

Seven of the nine parts of the 8th square, or a rectangle equal to 7-9ths of the 8th square, are greater than seven of the ten parts of the 9th square, or a rectangle equal to 7-10ths of the 9th square.

Eight of the ten parts of the 9th square, or a rect-

angle equal to 8-10ths of the 9th square, are greater than seven of the nine parts of the 8th square, or a rectangle equal to 7-9ths of the 8th square.

Eight of the nine parts of the 8th square, or a rectangle equal to 8-9ths of the 8th square, are greater than eight of the ten parts of the 9th square, or a rectangle equal to 8-10ths of the 9th square.

Nine of the ten parts of the 9th square, or a rectangle equal to 9-10ths of the 9th square, are greater than eight of the nine parts of the 8th square, or a rectangle equal to 8-9ths of the 8th square.

The whole of the 8th square is greater than 9-10ths of the 9th square, or a rectangle equal to 9-10ths of the 9th square.

The 9th square is equal to the 8th.

ONE of the ten equal parts of the 9th square, or a rectangle equal to 1-10th of the 9th square, is less than one of the nine equal parts of the 8th square, or a rectangle equal to 1-9th of the 8th square.

Two of the ten equal parts of the 9th square, or a rectangle equal to 2-10ths of the 9th square, are less than two of the nine equal parts of the 8th square, or a rectangle equal to 2-9ths of it.

Three of the ten parts of the 9th square, or a rectangle equal to 3-10ths of the 9th square, are less than three of the nine parts of the 8th square, or a rectangle equal to 3-9ths of it.

Four of the ten parts of the 9th square, or a rectangle equal to 4-10ths of the 9th square, are less than four of the nine parts of the 8th square, or than a rectangle equal to 4-9ths of it.

Five of the ten parts of the 9th square, or a rectangle equal to 5-10ths of the 9th square, are less than

five of the nine parts of the 8th square, or a rectangle equal to 5-9ths of it.

Six of the ten parts of the 9th square, or a rectangle equal to 6-10ths of the 9th square, are less than six of the nine parts of the 8th square, or a rectangle equal to 6-9ths of it.

Seven of the ten parts of the 9th square, or a rectangle equal to 7-10ths of the 9th square, are less than seven of the nine parts of the 8th square, or than a rectangle equal to 7-9ths of it.

Eight of the ten parts of the 9th square, or a rectangle equal to 8-10ths of the 9th square, are less than eight of the nine parts of the 8th square, or than a rectangle equal to 8-9ths of it.

Nine of the ten parts of the 9th square, or a rectangle equal to 9-10ths of the 9th square, are less than the whole of the 8th square.

And the two squares are equal.

ONE of the nine equal parts of the 8th square, or a rectangle equal to 1-9th of the whole square, is less than one of the eight equal parts of the 7th square, or than a rectangle equal to 1-8th of the square.

Two of the nine parts of the 8th square, or a rectangle equal to 2-9ths of the 8th square, are less than two of the eight parts of the 7th square, or than a rectangle equal to 2-8ths of it.

Three of the nine parts of the 8th square, or a rectangle equal to 3-9ths of the 8th square, are less than three of the eight parts of the 7th square, or than a rectangle equal to 3-8ths of it.

Four of the nine parts of the 8th square, or a rectangle equal to 4-9ths of the 8th square, are less than four of the eight parts of the 7th square, or than a rectangle equal to 4-8ths of it.

Five of the nine equal parts of the 8th square, or a rectangle equal to 5-9ths of the 8th square, are less than five of the eight equal parts of the 7th square, or than a rectangle equal to 5-8ths of the 7th square.

Six of the nine parts of the 8th square, or a rectangle equal to 6-9ths of the 8th square, are less than six of the eight parts of the 7th square, or than a rectangle equal to 6-8ths of it.

Seven of the nine parts of the 8th square, or a rectangle equal to 7-9ths of the 8th square, are less than seven of the eight parts of the 7th square, or than a rectangle equal to 7-8ths of it.

Eight of the nine parts of the 8th square, or a rectangle equal to 8-9ths of the 8th square, are less than the whole of the 7th square.

And the whole of the 8th square is equal to the whole of the 7th.

ONE of the eight equal parts of the 7th square, or a rectangle equal to 1-8th of the 7th square, is less than one of the seven equal parts of the 6th square, or a rectangle equal to 1-7th of the 6th square.

Two of the eight parts of the 7th square or a rectangle equal to 2-8ths of the 7th square, are less than two of the seven parts of the 6th square, or a rectangle equal to 2-7ths of it.

Three of the eight parts of the 7th square or a rectangle equal to 3-8ths of the 7th square, are less than three of the seven parts of the 6th square, or a rectangle equal to 3-7ths of it.

Four of the eight parts of the 7th square, or a rectangle equal to 4-8ths of the 7th square, are less than four of the seven parts of the 6th square, or a rectangle equal to 4-7ths of it.

Five of the eight equal parts of the 7th square or a rectangle equal to 5-8ths of the 7th square, are less than five of the seven equal parts of the 6th square, or a rectangle equal to 5-7ths of it.

Six of the eight parts of the 7th square, or a rectangle equal to 6-8ths of the 7th square, are less than six of the seven parts of the 6th square or a rectangle equal to 6-7ths of it

Seven of the eight parts of the 7th square or a rectangle equal to 7-8ths of the 7th square, are less than the whole of the 6th square;

And the whole of the 7th square is equal to the whole of the 6th.

ONE of the seven equal parts of the 6th square, or a rectangle equal to 1-7th of the 6th square, is less than one of the six equal parts of the 5th square, or a rectangle equal to 1-6th of the 5th square.

Two of the seven parts of the 6th square, or a rectangle equal to 2-7ths of the 6th square, are less than two of the six parts of the 5th square, or a rectangle equal to 2-6ths of it.

Three of the seven parts of the 6th square, or a rectangle equal to 3-7ths of the 6th square, are less than three of the six parts of the 5th square, or a rectangle equal to 3-6ths of it.

Four of the seven parts of the 6th square or a rectangle equal to 4-7ths of the 6th square, are less than four of the six parts of the 5th square, or a rectangle equal to 4-6ths of it.

Five of the seven parts of the 6th square, or a rectangle equal to 5-7ths of the 6th square, are less than five of the six parts of the 5th square, or a rectangle equal to 5-6ths of it.

Six of the seven equal parts of the 6th square or a rectangle equal to 6-7ths of the 6th square, are less than the whole of the 5th square;

And the whole of the 6th square is equal to the whole of the 5th square.

ONE of the six equal parts of the 5th square, or a rectangle equal to 1-6th of the 5th square, is less than one of the five equal parts of the 4th square or a rectangle equal to 1-5th of the 4th square.

Two of the six parts of the 5th square or a rectangle equal to 2-6ths of the 5th square, are less than two of the five parts of the 4th square, or a rectangle equal to 2-5ths of it.

Three of the six parts of the 5th square, or a rectangle equal to 3-6ths or the 4th square, are less than three of the five parts of the 4th square, or a rectangle equal to 3-5ths of it.

Four of the six parts of the 5th square, or a rectangle equal to 4-6ths of the 5th square, are less than four of the five parts of the 4th square, or a rectangle equal to 4-5ths of it.

Five of the six parts of the 5th square, or a rectangle equal to 5-6ths of the 5th square, are less than the whole of the 4th square.

The whole of the 5th square is equal to the whole of the 4th.

ONE of the five equal parts of the 4th square, or a rectangle equal to 1-5th of the 4th square, is less than one of the four parts of the 3d square or a rectangle equal to 1-4th of the 3d square.

Two of the five parts of the 4th square, or a rectangle equal to 2-5ths of the 4th square, are less than

two of the four parts of the 3d square, or a rectangle equal to 2-4ths of it.

Three of the five parts of the 4th square, or a rectangle equal to 3-5ths of the 4th square, are less than three of the four parts of the 3d square, or a rectangle equal to 3-4ths of it.

Four of the five parts of the 4th square, or a rectangle equal to 4-5ths of the 4th square, are less than the whole of the 3d.

The whole of the 4th square is equal to the whole of the 3d.

ONE of the four equal parts of the 3d square, or a rectangle equal to 1-4th of the 3d square, is less than one of the three equal parts of the second square, or a rectangle equal to 1-3d of the 2d square.

Two of the four parts of the 3d square, or a rectangle equal to 2-4ths of the 3d square, are less than two of the three parts of the 2d square, or a rectangle equal to 2-3ds of it.

Three of the four parts of the 3d square, or a rectangle equal to 3-4ths of the 3d square, are less than the whole of the 2d square.

The whole of the 3d square is equal to the whole of the 2d.

ONE of the three equal parts of the 2d square, or a rectangle equal to 1-3d of the 2d square, is less than one of the two equal parts of the 1st square, or a rectangle equal to one half of the 1st square.

Two of the three parts of the 2d square or a rectangle equal to 2-3ds of the 2d square, are less than the whole of the 1st square.

The whole of the 2d square is equal to the whole of the 1st square.

FOURTH EXERCISE.

THIRD LINE OF SQUARES.

FIRST SQUARE.

THIS square is divided by one horizontal line into two equal rectangles, and these two equal rectangles are also divided by one upright line into four lesser and equal squares.

SECOND SQUARE.

THIS square is divided by two horizontal lines into three equal rectangles, which are also divided by two upright lines into nine lesser and equal squares.

THIRD SQUARE.

THIS square is divided by three horizontal lines into four equal rectangles, which are also divided by three upright lines into sixteen lesser and equal squares.

FOURTH SQUARE.

THIS square is divided by four horizontal lines into five equal rectangles, which are also divided by four upright lines into twenty-five lesser and equal squares.

FIFTH SQUARE.

THIS square is divided by five horizontal lines

into six equal rectangles, which are also divided by five upright lines into thirty-six lesser & equal squares.

SIXTH SQUARE.

THIS square is divided by six horizontal lines into seven equal rectangles, which are also divided by six upright lines into forty-nine lesser and equal squares.

SEVENTH SQUARE.

THIS square is divided by seven horizontal lines into eight equal rectangles, which are also divided by seven upright lines into sixty-four lesser and equal squares.

EIGHTH SQUARE.

THIS square is divided by eight horizontal lines into nine equal rectangles, which are also divided by eight upright lines into eighty-one lesser and equal squares.

NINTH SQUARE.

THIS square is divided by nine horizontal lines into ten equal rectangles, which are also divided by nine upright lines into one hundred lesser and equal squares.

FIRST SQUARE.

TWO of these lesser squares taken horizontally adjacent to each other, form a half of the whole square, and also form a rectangle whose height is half its length.

Twice two of these lesser squares taken horizontally adjacent to each other, form the whole square.

SECOND SQUARE.

THREE of these nine lesser squares taken horizontally adjacent to each other, form a third part of the square, and also form a rectangle whose height is one-third of its length.

Twice three of these lesser squares taken horizontally adjacent, are two-thirds of the whole square, and form a rectangle whose height is equal to 2-3ds of its length.

Three times three of these lesser squares taken horizontally adjacent, form the whole second square.

THIRD SQUARE.

FOUR of these sixteen lesser squares taken horizontally adjacent to each other, form a fourth part of the square, and also form a rectangle, whose height is equal to one-fourth of its length.

Twice four of these lesser squares taken horizontally adjacent, are two-fourths of the whole square, and form a rectangle whose height is equal to two-fourths of its length

Three times four of these lesser squares taken, upright or horizontally adjacent, are three-fourths of the whole square, and form a rectangle whose height is equal to three-fourths of its length.

Four times four of these lesser squares taken horizontally adjacent, form the whole third square.

FOURTH SQUARE.

FIVE of these twenty-five lesser squares taken ho*rizontally* adjacent to each other form a fifth part of

the square, and also a rectangle whose height is equal to one-fifth of its length.

Twice five of these lesser squares, taken horizontally adjacent, are two-fifths of the whole square, and form a rectangle whose height is equal to two-fifths of its length.

Three times five of these lesser squares taken horizontally adjacent, are 3-5ths of the whole square, and form a rectangle whose height is equal to 3-5ths of its length.

Four times five of these lesser squares taken horizontally adjacent, are 4-5ths of the whole square, and form a rectangle whose height is equal to 4-5ths of its length.

Five times five of these lesser squares taken horizontally and vertically adjacent, form the whole 4th square.

FIFTH SQUARE.

SIX of these thirty-six lesser squares taken horizontally adjacent to each other, are a 6th part of the square, and form a rectangle whose height is equal to 1-6th of its length.

Twice six of these lesser squares taken horizontally adjacent, are 2-6ths of the whole square, and form a rectangle whose height is equal to 2-6ths of its length.

Three times six of these lesser squares taken horizontally adjacent, are 3-6ths of the whole square, and form a rectangle whose height is equal to 3-6ths of its length.

Four times six of these lesser squares taken hori-

zontally adjacent, are 4-6ths of the whole square, and form a rectangle whose height is equal to 4-6ths of its length.

Five times six of these lesser squares taken horizontally adjacent to each other, are 5-6ths of the whole square, and form a rectangle whose height is equal to 5-6ths of its length.

Six times six of these lesser squares taken horizontally and vertically adjacent, form the whole 5th square.

SIXTH SQUARE.

SEVEN of these forty-nine lesser squares taken horizontally adjacent to each other, are a 7th part of the square, and form a rectangle whose height is equal to 1-7th of its length.

Twice seven of these lesser squares taken horizontally adjacent, are 2-7ths of the whole square, and form a rectangle whose height is equal to 2-7ths of its length.

Three times seven of these lesser squares taken horizontally adjacent, are 3-7ths of the whole square, and form a rectangle whose height is equal to 3-7ths of its length.

Four times seven of these lesser squares taken horizontally adjacent, are 4-7ths of the whole square, and form a rectangle whose height is equal to 4-7ths of its length.

Five times seven of these lesser squares taken horizontally adjacent, are 5-7ths of the whole square, and form a rectangle whose height is equal to 5-7ths of its length.

Six times seven of these lesser squares taken hori-

zontally adjacent to each other, are 6-7ths of the whole square, and form a rectangle whose height is equal to 6-7ths of its length.

Seven times seven of these lesser squares taken horizontally and vertically adjacent, form the whole 6th square.

SEVENTH SQUARE.

EIGHT of these sixty-four lesser squares taken horizontally adjacent, are an 8th part of the square, and form a rectangle whose height is equal to 1-8th of its length.

Twice eight of these lesser squares taken horizontally adjacent, are 2-8ths of the whole square, and form a rectangle whose height is equal to 2-8ths of its length.

Three times eight of these lesser squares taken horizontally adjacent, are 3-8ths of the whole square, and form a rectangle whose height is equal to 3-8ths of its length.

Four times eight of these lesser squares taken horizontally adjacent, are 4-8ths of the whole square, and form a rectangle whose height is equal to 4-8ths of its length.

Five times eight of these lesser squares taken horizontally adjacent, are 5-8ths of the whole square, and form a rectangle whose height is equal to 5-8ths of its length.

Six times eight of these lesser squares taken horizontally adjacent, are 6-8ths of the whole square, and form a rectangle whose height is equal to 6-8ths of its length.

Seven times eight of these lesser squares taken horizontally adjacent, are 7-8ths of the whole square, and form a rectangle whose height is equal to 7-8ths of its length.

Eight times eight of these lesser squares taken horizontally and vertically adjacent, form the whole 7th square.

EIGHTH SQUARE.

NINE of these eighty-one lesser squares taken horizontally adjacent to each other, are a 9th part of the square, and form a rectangle whose height is equal to 1-9th of its length.

Twice nine of these lesser squares taken horizontally adjacent, are 2-9ths of the whole square, and form a rectangle whose height is equal to 2-9ths of its length.

Three times nine of these lesser squares taken horizontally adjacent, are 3-9ths of the whole square, and form a rectangle whose height is equal to 3-9ths of its length.

Four times nine of these lesser squares taken horizontally adjacent, are 4-9ths of the whole square, and form a rectangle whose height is equal to 4-9ths of its length.

Five times nine of these lesser squares taken horizontally adjacent, are 5-9ths of the whole square, and form a rectangle whose height is equal to 5-9ths of its length.

Six times nine of these lesser squares taken horizontally adjacent, are 6-9ths of the whole square, and form a rectangle whose height is equal to 6-9ths of its length.

Seven times nine of these lesser squares taken horizontally adjacent, are 7-9ths of the whole square, and form a rectangle whose height is equal to 7-9ths of its length.

Eight times nine of these lesser squares taken horizontally adjacent, are 8-9ths of the whole square, and form a rectangle whose height is equal to 8-9ths of its length.

Nine times nine of these lesser squares taken horizontally and vertically adjacent, form the whole 8th square.

NINTH SQUARE.

TEN of these one hundred lesser squares taken horizontally adjacent to each other, are 1-10th part of the square, and form a rectangle whose height is equal to 1-10th of its length;

Twice ten of these lesser squares taken horizontally adjacent, are 2-10ths of the whole square, and form a rectangle whose height is equal to 2-10ths of its length.

Three times ten of these lesser squares taken horizontally adjacent, are 3-10ths of the whole square, and form a rectangle whose height is equal to 3-10ths of its length.

Four times ten of these lesser squares taken horizontally adjacent, are 4-10ths of the whole square, and form a rectangle whose height is equal to 4-10ths of its length

Five times ten of these lesser squares taken horizontally adjacent, are 5-10ths of the whole square, and form a rectangle whose height is equal to 5-10ths of its length.

Six times ten of these lesser squares taken horizontally adjacent, are 6-10ths of the whole square, and form a rectangle whose height is equal to 6-10ths of its length.

Seven times ten of these lesser squares taken horizontally adjacent, are 7-10ths of the whole square, and form a rectangle whose height is equal to 7-10ths of its length.

Eight times ten of these lesser squares taken horizontally adjacent, are 8-10ths of the whole square, and form a rectangle whose height is equal to 8-10ths of its length.

Nine times ten of these lesser squares taken horizontally adjacent, are 9-10ths of the whole square, and form a rectangle whose height is equal to 9-10ths of its length.

Ten times ten of these lesser squares taken horizontally and vertically adjacent, form the whole ninth square.

3D SECTION.

FIRST SQUARE.

TWO of these four lesser squares taken vertically adjacent, are half of the whole square, and form a rectangle whose length is equal to half its height.

Twice two of these lesser squares taken vertically and horizontally adjacent, compose the whole square.

SECOND SQUARE.

THREE of these nine lesser squares taken vertically adjacent are a third part of the square, and

form a rectangle whose length is equal to a third part of its height.

Twice three of these lesser squares, taken vertically adjacent, are 2-3ds of the whole square, and form a rectangle whose length is equal to 2-3d parts of its height.

Three times three of these lesser squares taken vertically and horizontally adjacent, form the second square.

THIRD SQUARE.

FOUR of these sixteen lesser squares, taken vertically adjacent, are 1-4th of the square, and form a rectangle whose length is equal to 1-4th of its height.

Twice four of these lesser squares, taken vertically adjacent, are 2-4ths of the square, and form a rectangle whose length is 2-4ths of its height.

Three times four of these lesser squares, taken vertically adjacent, are 3-4ths of the square, and form a rectangle whose length is 3-4ths of its height.

Four times four of these lesser squares taken vertically and horizontally adjacent form the third square.

NOTE. It is evident that the lessons on the rectangles, composed of the lesser squares taken vertically, may be continued as in the case of their horizontal positions, without the details being farther repeated.

FIFTH EXERCISE.

A. A line or series of rectangles.

The height of the 1st rectangle is equal to half of its length.

The height of the 2d rectangle is equal to the 3d part of its length.

The height of the 3d rectangle is equal to the 4th part of its length.

The height of the 4th rectangle is equal to the 5th part of its length.

The height of the 5th rectangle is equal to the 6th part of its length.

The height of the 6th rectangle is equal to the 7th part of its length.

The height of the 7th rectangle is equal to the 8th part of its length.

The height of the 8th rectangle is equal to the 9th part of its length.

The height of the 9th rectangle is equal to the 10th part of its length.

1ST SECTION.

B. A line or series of straight lines which are neither upright nor horizontal; these lines are called Oblique or Slanting.

2D SECTION.

ALL these slanting lines are within their respective dotted rectangles.

THE 1st of these slanting lines is within a rectangle, whose height is equal to half its length.

The 2d slanting line is within a rectangle whose height is equal to the 3d part of its length.

The 3d slanting line is within a rectangle whose height is equal to the 4th part of its length.

The 4th slanting line is within a rectangle whose height is equal to the 5th part of its length.

The 5th slanting line is within a rectangle whose height is equal to the 6th part of its length.

The 6th slanting line is within a rectangle whose height is equal to the 7th part of its length.

The 7th slanting line is within a rectangle, whose height is equal to the 8th part of its length.

The 8th slanting line is within a rectangle, whose height is equal to the 9th part of its length.

The 9th slanting line is within a rectangle, whose height is equal to the 10th part of its length.

4TH SECTION.

ALL these slanting lines run from the left hand towards the right, in the rectangles in which they are contained; they are, therefore called "Lines slanting upwards from left to right."

5TH SECTION.

ALL these slanting lines begin from the left lower angle of the rectangle in which they are contained, and end at the right upper angle of the rectangle, cutting their respective rectangles from the left lower

angle to the right upper angle. They are, therefore, called " the Diagonal lines of the Rectangles, ascending from left to right.

6TH SECTION.

THE 1st of these slanting lines is a diagonal, ascending from left to right in a rectangle, whose height is half its length.

The 2d of these slanting lines is a diagonal ascending from left to right, in a rectangle, whose height is a third part of its length.

The 3d of these slanting lines is a diagonal, ascending from left to right in a rectangle, whose height is a fourth part of its length.

The 4th of these slanting lines is a diagonal ascending from left to right in a rectangle, whose height is a fifth part of its length.

The 5th of these slanting lines is a diagonal ascending from left to right in a rectangle, whose height is a sixth part of its length.

The 6th of these slanting lines is a diagonal, ascending from left to right in a rectangle, whose height is a seventh part of its length.

The 7th of these slanting lines is a diagonal, ascending from left to right in a rectangle, whose height is an eighth part of its length.

The 8th of these slanting lines is a diagonal, ascending from left to right in a rectangle, whose height is a ninth part of its length.

The 9th of these slanting lines is a diagonal, ascending from left to right in a rectangle, whose height is a tenth part of its length.

1ST SECTION.

C. A line, or series of diagonals.

2D SECTION.

EACH of these diagonals ascend from the right hand towards the left; they are, therefore, called "Diagonals, ascending or slanting upwards to the left."

The

3D SECTION.

EACH of these slanting lines ascends with the same inclination from the right to the left, as the diagonal immediately above it did from left to right, only in opposite directions.

4TH SECTION.

THE 1st of these diagonals slants upwards from right to left with the same inclination that the diagonal immediately above it slants upwards from left to right in a rectangle, whose height is half its length.

The 2d of these diagonals slants upwards from right to left with the same inclination that the diagonal immediately above it slants upwards from left to right in a rectangle, whose height is one-third of its length.

The 3d of these diagonals slants upwards from right to left with the same inclination that the diagonal immediately above it slants upwards from left to right in a rectangle, whose height is one-fourth of its length.

The 4th of these diagonals slants upwards from right to left, with the same inclination that the diago-

nal immediately above it slants upwards from left to right, in a rectangle whose height is one-fifth of its length.

The 5th of these diagonals slants upwards from right to left, with the same inclination that the diagonal immediately above it slants upwards from left, to right, in a rectangle whose height is one-sixth of its length.

The 6th of these diagonals slants upwards from right to left, with the same inclination that the diagonal immediately above it slants upwards from left to right in a rectangle whose height is one-seventh of its length.

The 7th of these diagonals slants upwards from right to left with the same inclination that the diagonal immediately above it slants upwards from left to right in a rectangle whose height is one-eighth of its length.

The 8th of these diagonals slants upwards from right to left with the same inclination that the diagonal immediately above it slants upwards from left to right in a rectangle whose height is one ninth of its length.

The 9th of these diagonals slants upwards from right to left, with the same inclination that the diagonal immediately above it slants upwards from left to right, in a rectangle whose height is one-tenth of its length.

D. A line or series of rectangles.

The length of the 1st rectangle is half of its height.

The length of the 2d rectangle is one-third of its height.

The length of the 3d rectangle is one-fourth of its height.

The length of the 4th rectangle is one-fifth of its height.

The length of the 5th rectangle is one-sixth of its height.

The length of the 6th rectangle is one-seventh of its height.

The length of the 7th rectangle is one-eighth of its height.

The length of the 8th rectangle is one-ninth of its height.

The length of the 9th rectangle is one-tenth of its height.

1ST.

E. A series of diagonals, slanting upwards alternately to the right and to the left.

2ND.

The diagonals, slanting upwards to the right, are placed within dotted rectangles.

3D

The first of these lines is a diagonal, slanting upwards to the right, in a rectangle, whose length is half its height

The second of these lines slants upwards from right to left, in the contrary direction, but with the same inclination as a diagonal in a rectangle, whose length is half its height.

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Roundwood, Co. Wicklow.

APPENDIX.

IN order to exemplify the practical use of the foregoing Exercises, it has been deemed expedient to subjoin a few specimens of visible objects, to be first produced by the Pupils upon their Slates, from the dictation here given; and they should afterwards be required to produce in writing a similar description, the object only being submitted to their observation; it is evident that the objects must, as yet, be limited to such as are expressed either by Straight or Curved lines, (being parts of a circle,) or both combined together; and it is hoped that the specimens here given will be sufficient to enable intelligent teachers to prepare others for themselves.

It is also to be remembered, that strict mathematical precision is not aimed at, but only such accuracy as is sufficient to enable the young mind to reason with itself as to the proportions of any object it desires to observe for the purpose of describing its outline.

FIG. I.

(See Appendix Plate.)

Draw a Horizontal Straight line.

Divide it by nine points into ten equal parts.

From each point of division, and also at the right and left extremity of the Horizontal line, draw an Upright Straight line equal to half a tenth.

Join the upper extremities of these Upright Straight lines by a Horizontal Straight line.

Cut off half a tenth of that line at either extremity.

Divide the intermediate part by eight points into nine equal parts.

From each point of division, and also from the points of division that cut off the half-tenth, draw Upright Straight lines equal to half a tenth.

Join the upper extremities of these Upright Straight lines by a Horizontal Straight line.

Cut off half a tenth of that line at either extremity.

Divide the intermediate part by seven points into eight equal parts.

From each point of division, and also from the points that cut off half a tenth, draw Upright Straight lines equal to half a tenth.

Join the upper extremities of these Upright Straight lines by a Horizontal Straight line.

Cut off half a tenth of that line at either extremity.

Divide the intermediate part by six points into seven equal parts.

From each point of division, and also from the points that mark off half a tenth, draw Upright Straight lines equal to half a tenth.

Join the upper extremities of these Upright lines by a Horizontal Straight line.

Cut off half a tenth of that line at either extremity.

Divide the intermediate part by five points into six equal parts.

From each point of division, and also from the points that mark of half a tenth, draw Upright Straight lines equal to half a tenth.

Join the upper extremities of these Upright Straight lines by a Horizontal Straight line.

Cut off half a tenth of that line at either extremity.

Divide the intermediate part by four points into five equal parts.

From each point of division, and also from the points that mark off half a tenth, draw Upright Straight lines equal to half a tenth.

Join the upper extremities of these Upright straight lines by a Horizontal straight line.

Cut off half a tenth of that line at either extremity.

Divide the intermediate part by three points into four equal parts.

From each point of division, and also from the points that mark off half a tenth, draw Upright Straight lines equal to half a tenth.

Join the upper extremities of these Upright straight lines by a Horizontal Straight line.

Cut off half a tenth of that line at either extremity.

Divide the intermediate part by two points into three equal parts.

From each point of division, and also from the points that mark off half a tenth, draw Upright straight lines equal to half a tenth.

Join the upper extremities of these Upright Straight lines by a Horizontal Straight line.

Cut off half a tenth of that line at either extremity.

Divide the intermediate part by one point into two equal parts.

From each point of division, and also from the points that mark off half a tenth, draw an Upright straight line equal to half a tenth.

Join the upper extremities of these Upright lines by a Horizontal Straight line.

Cut off half a tenth at either extremity of that line.

Draw an Upright Straight line from each point that cuts off half a tenth.

Join the upper extremities of these Upright lines by a Horizontal Straight line.

FIG. II.

(See Appendix Plate.)

Draw a Straight line slanting downwards from left to right, as the diagonal of a rectangle whose length is equal to one-third of its height.

From the upper extremity of this line, draw a Straight line slanting downwards from right to left, and having the same inclination as the foregoing one; observing to make the last drawn line *strong* and the former *fine*, or what is usually called a *hair-stroke*.

Divide these Slanting lines by one point into two equal parts; and draw a Horizontal line joining the points of division.

Join the fifth point of division in the second half of the line, to the second point of division in the first half of the line, by a half circle bent to the left.

Join the second point of division in the first half of the line, to the sixth point of division in the second half of the line, by a half circle bent to the right.

Join the sixth point of division in the second half of the line, to the first point of division in the first half of the line, by a half circle bent to the left.

Join the first point of division in the first half of the line, to the seventh point of division in the second half of the line, by a half circle bent to the right.

Join the seventh point of division in the second half, to the upper extremity of the upright line, by a half circle bent to the left.

Join the upper extremity of the first half of the Upright line to the lower extremity by a half circle bent to the right.

NOTE.—The Pupil having once described this figure correctly, from dictation, it will be excellent practice to begin at the interior point, and describe this curve at once, both from left to right and from right to left; then beginning at the exterior point, to describe it inwards, in both directions. He may then learn to designate this Figure by the name of *A Circular Curved Line winding outwards* (or inwards when so drawn) *from right to left*, (or left to right,) as directed, as many times as there are pairs of half circles, each pair denoting, as it were, one revolution of the curve, as the figure in the plate does *seven times and a half.*

FIG. IV.

(See Appendix Plate.)

Draw a Horizontal Straight line.

Divide it by four large points into five equal parts.

Divide the two first fifths, each by one lesser point into two equal parts.

Join the beginning of the line to the point dividing the second fifth, by a half circle bent upwards.

Join the point dividing the second fifth to the point dividing the first fifth, by a half circle bent downwards.

Join the beginning of the line to the second large point of division, by a half circle bent downwards.

Join the second large point of division to the end of the line, by a half circle bent upwards.

Join the third large point of division to the end of the line by a half circle bent downwards,

Join the third large point of division to the 4th large point of division, by a half circle bent upwards.

NOTE.—This Figure, like the former, after having been once correctly drawn from dictation, should be practised at once, beginning from either extremity of the curve, and directing each half circle to be drawn *strong* in the middle and *fine* at the points. It will be at the same time an excellent preparative for writing and much improve the symmetry of the figure. In this and the preceding Figures, it is evident that the Straight Lines are only used as scales; in all such cases they should be drawn as light as possible, and erased as soon as the figure is completed; they are, therefore, expressed by dotted lines. It is hoped, after a little experience, the Pupils will be enabled to suppose them applied to objects without being at the pains of drawing them.

Join this point to the left extremity of the curved line bent downwards, by a curved line bent to the left.

Join this point to the right extremity of the curved line bent downwards by a curved line bent to the right.

NOTE.—It is evident this Figure will be much embellished by the finer lines given in the plate, and these are dictated by directing points to be marked in the several curved lines, and these points to be joined by curved lines having the direction of those in the figure.

FIG. VII.

(See Appendix Plate.)

Draw a Curved line bent upwards.

Join the extremities of it by a curved line bent downwards below it.

From either extremity where the curved lines meet, draw an Upright Straight line below it, equal to a Straight line which would join the extremities of the curves.

Join the lower extremities of these Upright lines by a curved line bent downwards.

At a small distance from this curve draw another parallel to it.

Cut off by a point two-thirds of the Upright line next the right hand.

Join the beginning of the line to the second point of division by a half circle bent to the right.

Draw another curve at a small distance from it, parallel to it.

FIG. VIII.

(See Appendix Plate.)

Draw a *light* Horizontal Straight line.

Divide it by one point into two equal parts.

From the point of division draw a Straight line slanting upwards from left to right, whose inclination is equal to that of the diagonal of a whole square, and equal in length to the whole of the Horizontal line.

From the same point draw another Straight line, slanting upwards from right to left, having the same inclination as the former, but in the contrary direction, and equal to it in length.

Across the upper extremity of the first drawn slanting line draw a straight line, equal and parallel to the second drawn slanting line, half at the right and half at the left side of the slanting line.

Through the upper extremity of the second drawn Slanting line, draw another Straight line equal and parallel to the first drawn Slanting line, and so situated

that half be at the right side and half at the left of the slanting line.

Join the point of division of the base line to the upper extremity of the first drawn slanting line, by a Curved line *(or fourth part of a Circle)* bent upwards, inclining to the left; and at a very small distance above it, draw another Curved line parallel to it.

Join both extremities of the second drawn slanting line with curved lines in like manner, bent upwards inclining to the right.

Join the beginning of the base line to the point of division, by a curved line *(or fourth part of a circle)* bent upwards.

Join the same point by a similar Curved line bent downwards.

Join the point of division to the end of the base line in like manner, both by a curved line bent upwards and one bent downwards.

Join the upper extremity of the first drawn parallel line to its middle point by a curved line bent upwards, inclining to the right, and also by a curved line bent downwards inclining to the left.

Join the middle point of the same line to its lower extremity, by a curved line bent upwards, inclining to the right, and also by a curved line bent downwards *inclining* to the left.

Join the upper extremity of the second drawn straight line to its middle point by a curved line bent upwards, inclining to the left, and also by a curved line bent downwards inclining to the right.

Join the middle point of the same line to its lower extremity, by a curved line bent upwards, inclining to the left, and also by a curved line bent downwards inclining to the right.

From the point of division of the Horizontal line draw an Upright line below it, equal to one-third of it.

Join the extremities of this last line towards the right hand by a curved line bent to the right.

At a small distance from this curved line draw another parallel to it.

Rub out all the Straight lines.

FIG. IX.

(See Appendix Plate)

Draw a Horizontal Straight line.

At each extremity of it draw an Upright Straight line equal to twice the length of the Horizontal line.

Join the upper extremities of these Upright lines by a Horizontal straight line.

Divide the Horizontal sides of this rectangle by three points into four equal parts.

Divide the Upright sides of this rectangle by seven points into eight equal parts.

At the distance of one-fourth of the base line, outside the rectangle, draw lines parallel to the sides of the rectangle, and with these lines complete a rectangle exterior to the first.

Upon the right upright side of this exterior rectangle draw another rectangle equal and similar to it.

At the distance of one-fourth within this, describe another rectangle equal and similar to the first.

Divide the Horizontal sides of this rectangle by three points into four equal parts.

Divide the Upright sides of this rectangle by seven points into eight equal parts.

From the first point of division on the left Upright side to the first point on the upper Horizontal side, draw a Slanting line.

From the second point on the left Upright side to the second point on the upper Horizontal side, draw a Slanting line.

From the third point on the left Upright side to the third point on the upper Horizontal side, draw a Slanting line.

From the fourth point on the left Upright side to

the right extremity of the upper Horizontal side, draw a Slanting line.

From the fifth point on the left Upright side to the first point on the right Upright side, draw a Slanting line.

From the sixth point on the left Upright side to the second point of the right Upright side, draw a Slanting line.

From the seventh point on the left Upright side to the third point on the right Upright side, draw a Slanting line.

From the lower extremity of the left Upright side to the fourth point on the right Upright side, draw a Slanting line.

From the first point on the lower Horizontal side to the fifth point on the right Upright side, draw a Slanting line.

From the second point on the lower Horizontal side to the sixth point on the right Upright side, draw a Slanting line.

From the third point on the lower Horizontal side to the seventh point on the right Upright side draw a Slanting line.

From the first point of division on the right Upright side to the third point on the upper Horizontal side, draw a Slanting line.

From the second point on the right Upright side to the second point on the upper Horizontal side, draw a Slanting line.

From the third point on the right Upright side to the first point on the upper Horizontal side, draw a Slanting line.

From the fourth point on the right Upright side to the left extremity of the upper Horizontal line, draw a Slanting line.

From the fifth point on the right Upright side to the first point on the left upright side, draw a Slanting line.

From the sixth point on the right upright side to the second point on the left upright side, draw a Slanting line.

From the seventh point on the right upright side to the third point on the left upright side, draw a Slanting line.

From the lower extremity of the right upright side to the fourth point on the left upright side, draw a Slanting line.

From the first point of division on the lower Horizontal side to the fifth point on the left upright side, draw a Slanting line.

From the second point on the lower Horizontal

side to the sixth point on the left upright side, draw a Slanting line.

From the third point on the lower Horizontal side to the seventh point on the left Upright side, draw a Slanting line.

NOTE.—It is evident that the points of division of the first drawn interior rectangle may be joined in like manner, in order to complete the object as in the Plate.

FIG. X.

(See Appendix Plate.)

Draw a Horizontal Straight line.

Divide it by nine points into ten equal parts.

At each extremity of it draw an upright line above it, equal to three-tenths.

Above the third point of division draw an upright line equal to three-tenths.

Above the seventh point of division draw an upright line equal to three-tenths.

Join the upper extremity of the first and second Upright lines with a horizontal line.

Join also the third and fourth with a horizontal line.

Draw Horizontal lines at a small distance above these, equal and parallel to them.

Mark a point above the fifth point of division at the height of five-tenths from it.

Join this point by a Slanting line to the right upper angle of the square next the left.

Draw another equal and parallel to it, at a small distance above it.

And also by another Slanting line to the left upper angle of the square next the right.

Draw another equal and parallel to it at a small distance above it.

From the top of the left upper angle of the left square draw a line slanting upwards, parallel to the first drawn slanting line to the perpendicular height of four-tenths and a half above the base line, and draw a horizontal line from its upper extremity towards your right hand, until it meets the first drawn slanting line.

From the top of the right upper angle of the right square draw a line slanting upwards, parallel to the second drawn slanting line, to the perpendicular height of four tenths and a half above the base line, and draw a horizontal line from its upper extremity towards your left hand until it meets the second drawn slanting line.

Above the second tenth, at the height of one-tenth, draw a horizontal line equal and parallel to it, and upon this line complete a square above it.

Divide each of its sides by three points into four equal parts.

Join the first point of the upper horizontal side to the first point of the left upright side by a slanting line.

Join the second point of the upper horizontal side to the second of the left upright side, by a slanting line.

Join the third point of the upper horizontal side to the third point of the left upright side by a slanting line.

Join the first of the right upright side to the first of the lower horizontal side.

Join the second of the right upright side to the second of the lower horizontal side.

Join the third of the right upright side to the third of the lower horizontal side.

Join also the third of the upper horizontal side by a slanting line to the first of the right upright side.

Join the second of the upper horizontal side to the second of the right upright side.

Join the third of the upper horizontal side to the third of the right upright side.

Join the first of the left upright side to the third of the lower horizontal side.

Join the second of the left upright side to the second of the lower horizontal side.

Join the third of the left upright side to the first of the lower horizontal side.

Above the ninth-tenth, at the height of one-tenth, draw a horizontal line equal and parallel to it, and upon this line complete a square above it.

Divide each of its extremities and draw slanting lines as in the square, above the second tenth.

Above the fifth and sixth tenths, at the height of three-tenths, draw a horizontal straight line equal to two-tenths.

Join its extremities by a half circle bent upwards.

Divide this horizontal line by one point into two equal parts.

From the point of division draw an Upright line upwards until it meets the half circle.

Divide this upright line by three points into four equal parts.

Divide each of the parts of the horizontal line by three points into four equal parts.

Join the third point of division of the first half of the horizontal line, to the third point of division of the upright side, by a slanting line, and extend it upwards to the right until it meets the half circle.

In like manner the remaining points may be suc-

cessively directed to be joined until all the lines in the Plate are drawn.

From the fourth and sixth points of division of the base line draw upright straight lines above it, equal to two-tenths of the base.

Join their upper extremities by a curved line bent upwards.

Divide the fifth and sixth tenths of the base respectively, by five points into six equal parts.

Draw upright lines through each of the points of division upward, until they meet the half circle ; and strengthen the one which passes through the fifth large point of division of the base.

NOTE. It is evident that this Figure, when compleated, may give rise to an infinite number of useful calculations, such as, "Supposing the base line to be 40 feet long, how many feet of surface does the whole front contain?" "How many perches of Mason-work?" "How many square feet in the doors, windows, &c. &c.?"

FIG. XL.

(See Appendix Plate.)

Draw a Horizontal Straight line.

Divide it by three points into four equal parts.

From the left extremity of this line draw an upright line equal to three-fourths of it.

From the first point of division also draw an upright line equal to three-fourths.

From the end of the line draw an upright line equal to one-fourth and half a fourth.

From the second upright line cut off a part equal to the third upright line.

Join the point of division to the upper extremity of the third upright line, by a horizontal straight line.

Divide this line by eight points into nine equal parts; and again subdivide these parts each by one point into two equal parts.

Join the upper extremities of the first and second upright lines by a horizontal straight line.

Extend this line to a very small distance beyond the upright lines at either extremity.

Draw another equal and parallel to it, at a small distance above it

Join the corresponding extremities of these two lines by short upright lines, and divide the uppermost of the horizontal lines by six points into seven equal parts.

At the distance of one-third of a seventh above the second seventh draw a horizontal line, equal and parallel to the second seventh.

Do the same above the fourth and sixth-sevenths.

From each extremity of these lines, above them, draw upright straight lines equal to one-seventh.

From the beginning and end of the horizontal

line, and above it, draw Upright Straight lines, equal to one-seventh and one-third of a seventh.

You will then have eight of these upright lines.

Join the upper extremity of the first to the upper extremity of the second, by a horizontal straight line; and draw another horizontal line at a small distance above it, equal and parallel to it.

Join the upper extremities of the third and fourth lines by a horizontal straight line, and draw another at a small distance above it, equal and parallel to it.

Join the fifth to the sixth by a horizontal straight line, and draw another at a small distance above it, equal and parallel to it.

Join the seventh upright line to the eighth by a horizontal line, and draw another horizontal line at a small distance above it, equal and parallel to it.

From the lower part of the second principal upright line cut off a part equal to two-fourths and half a fourth of the base line.

From that point draw a horizontal line towards the right hand, equal to two-fourths of the base line.

Join the right extremity of this horizontal line to upper extremity of the third upright line, by a straight line slanting downwards from left to right.

From the end of the line draw an upright line equal to one-fourth and half a fourth.

From the second upright line cut off a part equal to the third upright line.

Join the point of division to the upper extremity of the third upright line, by a horizontal straight line.

Divide this line by eight points into nine equal parts; and again subdivide these parts each by one point into two equal parts.

Join the upper extremities of the first and second upright lines by a horizontal straight line.

Extend this line to a very small distance beyond the upright lines at either extremity.

Draw another equal and parallel to it, at a small distance above it.

Join the corresponding extremities of these two lines by short upright lines, and divide the uppermost of the horizontal lines by six points into seven equal parts.

At the distance of one-third of a seventh above the second seventh draw a horizontal line, equal and parallel to the second seventh.

Do the same above the fourth and sixth-sevenths.

From each extremity of these lines, above them, draw upright straight lines equal to one-seventh.

From the beginning and end of the horizontal

line, and above it, draw Upright Straight lines, equal to one-seventh and one-third of a seventh.

You will then have eight of these upright lines.

Join the upper extremity of the first to the upper extremity of the second, by a horizontal straight line; and draw another horizontal line at a small distance above it, equal and parallel to it.

Join the upper extremities of the third and fourth lines by a horizontal straight line, and draw another at a small distance above it, equal and parallel to it.

Join the fifth to the sixth by a horizontal straight line, and draw another at a small distance above it, equal and parallel to it.

Join the seventh upright line to the eighth by a horizontal line, and draw another horizontal line at a small distance above it, equal and parallel to it.

From the lower part of the second principal upright line cut off a part equal to two-fourths and half a fourth of the base line.

From that point draw a horizontal line towards the right hand, equal to two-fourths of the base line.

Join the right extremity of this horizontal line to upper extremity of the third upright line, by a straight line slanting downwards from left to right.

Divide the part of the second upright line intercepted between the two points of division, from whence the horizontal lines have been already drawn towards the right, by nine points into ten such parts that the first shall be less than the second; the second than the third; the third than the fourth; the fourth than the fifth; the fifth than the sixth; the sixth than the seventh; the seventh than the eighth; the eighth than the ninth; the ninth than the tenth.

Through each of these points draw horizontal lines towards the right, until they meet the slanting line.

You have then eleven horizontal lines.

From the points of division of the eleventh line draw upright lines upwards till they meet the tenth horizontal line, which will thereby be divided into eighteen parts.

Mark the points which would divide all the parts of this line, except the last, into halves, and from these 17 points draw upright lines upwards, till they meet the ninth horizontal line, which will thereby be divided by 17 points into 18 parts; and mark the points which would divide all these parts, except the first and last, into halves.

Draw upright lines from these points, upwards, till they meet the eighth horizontal line, which will *thereby* be divided by 15 points into 16 parts;

mark the points which would divide all the parts except the last, into halves.

Draw upright lines from these points upwards till they meet the seventh, which will thereby be divided by 15 points into 16 parts; mark the points that would divide all the parts except the first and last, into halves.

Draw upright lines from these points upwards till they meet the sixth horizontal line, which will thereby be divided by 14 points into 15 parts, and mark the points that would divide all the parts except the last, into halves.

Draw upright lines from these points upwards till they meet the fifth horizontal line, which will thereby be divided by 14 points into 15 parts, and mark all the points that would divide all the parts except the first and last, into halves.

Draw upright lines from these points upwards till they meet the fourth horizontal line which will thereby be divided by 13 points into 14 equal parts, and mark the points that would divide all the parts except the last, into halves.

Draw upright lines from these points upwards till they meet the third horizontal line, which will thereby be divided by 13 points into 14 parts, and mark the points that would divide all the parts, except the first and last, into halves.

From these points draw upright lines upwards till they meet the second horizontal line, which will thereby be divided by 12 points into 13 parts, and mark the points that would divide all the parts, except the last, into halves.

From these points draw upright lines upwards till they meet the first horizontal line.

Below the eleventh line draw a horizontal line at a small distance from it, equal and parallel to it.

Above the horizontal line, at a small distance from it, draw a horizontal line equal and parallel to it.

Divide each of the fourths of the original base line by two points into three equal parts.

From each of the points of division of the first fourth draw upright straight lines equal to half a fourth.

Mark also in the base line the point that would divide the first fourth into two equal parts.

Also above this point, at the distance of three fourths of a fourth of the base, mark a point.

Join this point to the upper extremity of the upright line next the left hand, by a curved line bent to the left, inclining upwards.

Join it also to the upper extremity of the upright

line next the right hand, by a curved line bent to the right, inclining upwards.

Outside these curved lines, at a small distance from them, draw curved lines parallel to each of them, and join their lower extremities to the base line by upright straight lines.

Divide the second third of the first fourth of the base line by five points into six equal parts.

From these points draw upright lines till they meet the curves above them.

Divide each of the fourths of the original base line by three points into four equal parts, taking care to distinguish these points from those which divide the parts of the base into three equal parts.

At the height of one-fourth of the base line, above the second and third fourths of the first part of it, draw a horizontal straight line equal to two-fourths of that part

From each extremity of this line draw upright lines above it, equal to one-fourth of the same part of the base.

Join the upper extremities of these upright lines by a horizontal straight line.

Above the middle third of the first part of the base line, and at the height of two-fourths of the base, draw a horizontal straight line, equal and parallel to the middle third.

From each extremity of it draw upright lines upwards, equal to one half of the first part of the base line, and at a small distance outside each of these draw upright lines parallel to them.

Extend the last drawn horizontal line to meet these latter upright lines, and likewise a small portion beyond them at either extremity.

Below this last horizontal line, at a small distance from it, draw a horizontal line equal to it, when extended.

Join their corresponding ends by upright straight lines.

Divide the uppermost of these horizontal lines also, by one point into two equal parts.

Above this point mark another at the distance of three-fourths of the first part of the base.

Join this point to the upper extremity of the interior upright line towards the right hand, by a curved line bent upwards inclining to the right.

From this point draw another curved line bending upwards, inclining to the left, to the upper extremity of the interior upright line, towards the left hand.

Draw other curved lines at small distances without *these*, parallel to them.

Join the point dividing the uppermost of the two last drawn horizontal lines to the point where the two curves meet, by upright straight lines at small distances from one another.

Divide the one next the right by nine points into ten equal parts, and from these points draw horizontal lines towards the right hand, till they meet the upright line, and the curve above it.

Divide the one next the left hand by eight points into nine equal parts, and from these points draw horizontal lines towards the left hand, till they meet the upright line, and the curve above it.

Of the four lesser horizontal lines at the top of the figure, divide the first and fourth each by one point into two equal parts.

From these points draw upright lines upwards equal to twice their respective horizontal lines.

Join the upper extremities of these upright lines to both extremities of their respective horizontal lines by slanting lines.

Immediately above the middle point of the first fourth of the base and at the height of five-fourths and a half, mark a point.

Join this point to the left extremity of the short horizontal line immediately above the first point of division of the horizontal line, which has been already divided into seven equal parts.

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Join this point also to the right extremity of the short horizontal line, immediately above the sixth point of division of the same line.

Join it also to the point that would divide the second of these lesser horizontal lines into two equal parts.

Through this upper point draw a short upright line and across its middle point draw a short horizontal line both ways.

Above the second and third fourths of the second part of the base line, and at the height of one-fourth, draw a horizontal straight line equal to two-fourths, of one part of the base.

Divide it by three points into four equal parts.

From either extremity of this horizontal line draw upright lines equal to three-fourths of one part of the base.

Above the second point of division of the last drawn Horiontal line, at the height of twice and one-fourth of itself, mark a point.

Join this point to the upper extremity of the right upright line by a curved line bent upwards inclining to the right.

Join this point also to the upper extremity of the left upright line by a curved line bent upwards inclining to the left.

Draw curved lines parallel to these at a small dist-

ance without them, and also lines parallel to the upright lines, at the same distance without them.

Extend the base line a small distance at either extremity. From each extremity of it draw short upright lines below it, and join their lower extremities by a horizontal line.

From each of the three points of division of the extended horizontal line, draw upright lines equal to those already drawn.

Join the upper extremities of these upright lines by a horizontal line drawn across until it meets the lower extremities of the curves.

Divide these upright lines by three points into four equal parts.

Through the points of division draw horizontal lines extending from the left upright to the right upright line.

Join the upper extremity of the left upright line to the upper extremity of the third upright line, by a half circle bent upwards.

Join the upper extremity of the third upright line to the upper extremity of the right upright line, by a half circle bent upwards.

Above these half circles describe a whole circle as large as can be contained between them and the upper curved lines.

Join this point also to the right extremity of the short horizontal line, immediately above the sixth point of division of the same line.

Join it also to the point that would divide the second of these lesser horizontal lines into two equal parts.

Through this upper point draw a short upright line, and across its middle point draw a short horizontal line both ways.

Above the second and third fourths of the second part of the base line, and at the height of one-fourth, draw a horizontal straight line equal to two-fourths, of one part of the base.

Divide it by three points into four equal parts.

From either extremity of this horizontal line draw upright lines equal to three-fourths of one part of the base.

Above the second point of division of the last drawn Horiontal line, at the height of twice and one-fourth of itself, mark a point.

Join this point to the upper extremity of the right upright line by a curved line bent upwards inclining to the right.

Join this point also to the upper extremity of the left upright line by a curved line bent upwards inclining to the left.

Draw curved lines parallel to these at a small dist-

ance without them, and also lines parallel to the upright lines, at the same distance without them.

Extend the base line a small distance at either extremity. From each extremity of it draw short upright lines below it, and join their lower extremities by a horizontal line.

From each of the three points of division of the extended horizontal line, draw upright lines equal to those already drawn.

Join the upper extremities of these upright lines by a horizontal line drawn across until it meets the lower extremities of the curves.

Divide these upright lines by three points into four equal parts.

Through the points of division draw horizontal lines extending from the left upright to the right upright line.

Join the upper extremity of the left upright line to the upper extremity of the third upright line, by a half circle bent upwards.

Join the upper extremity of the third upright line to the upper extremity of the right upright line, by a half circle bent upwards.

Above these half circles describe a whole circle as large as can be contained between them and the upper curved lines.

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upper extremity of the upright line by
bent downwards inclining to the right.
at the upper extremity of the upright
right extremity of the horizontal line by a
bent downwards inclining to the left.
point at the right extremity of the hori-
to the lower extremity of the upright line,
ved line bent upwards inclining to the left.
point at the lower extremity of the upright
to the left extremity of the horizontal line, by
ved line bent upwards inclining to the right.

On either sides of the middle upright line draw upright lines parallel to it, at small distances from it.

NOTE. In like manner the remaining windows may be described above the other portions of the base line, and it is evident that the effect of stone work might be produced on any scale by applying the directions given in *FIG. I.*

FIG. XII.

(See Appendix Plate.)

Draw a half circle bent upwards.

Join its extremities *(lightly)* with a horizontal line.

Continue the curve of the circle from the left extremity to a small distance below it.

Divide the horizontal line by three points into four equal parts.

Below the first and second points of division, on a line with the lower extremity of the curved line when extended, mark two points.

Join the first of these points to the second by a curved line bent upwards.

From the second of these points draw an upright line downwards, equal to the horizontal line.

From the first of these points draw an upright line downwards, equal to half the horizontal line, and draw another parallel to it at a small distance from it to the left; and from the left extremity of the original curve, when extended, draw another upright line equal to these.

Divide the greater upright line by one point into two equal parts.

To the left of the line, at a small distance below the point of division, mark a point.

Join this point to the lower extremities of each of the upright lines by curved lines bent downwards, inclining to the left.

From the right extremity of the half circle draw an upright line downwards, until its lower extremity

ranges with the lower extremity of the greater upright line already drawn.

From the beginning of the horizontal line draw a straight line slanting upwards from left to right, having the same inclination as the diagonal of one-third of a square placed horizontally.

Divide this slanting line by three points into four equal parts.

Divide the second and third fourths each by one point into two equal parts.

Join these two latter points by a curved line bent upwards, inclining to the left, and also by a curved line bent downwards, inclining to the right.

Within these two curved lines describe a small circle, and above the curved line bent upwards, draw another parallel to it at a small distance from it.

Rub out the horizontal and slanting lines.

ERRATA.

Description of Forms.

Page 5, Line 12, for "*What do mean,*" read, "*What do you mean.*"

31, Line 13, for "*less,*" read "*greater.*"

35, Line 1, for "*below,*" read *to the left of,*" &c.

36, Last line but two, insert, "*Divide each fourth by one point into two equal parts.*"

Appendix.

Page 5, Line 15, for the word "*Strong,*" read "*Fine, or what is usually called, a hair-stroke.*"

Line 16, Instead of "*Fine,*" read "*Strong,*" (expressed in the Plate by a double line.)

6, After the fourth line insert, "*From the beginning and end of the line, and from the point of division, draw horizontal lines towards the right, equal to one-third of the upright lines.*"

8, Line 18, Insert, "*of the second half,*" after the words "*lower extremity.*"

11, Last line but one, for "*three times half itself,*" read, "*equal to the horizontal line.*"

21, Fifth line from the bottom, after the word "*downwards*", add, "*draw another Curved line below it, parallel to it.*"

15, Tenth line, For "*one-third,*" read "*one-half.*"

16, Eleventh line, for "*At the distance,*" read, *at the same distance as before.*"

21, After the ninth line, insert, "*Join the right extremity of the upper horizontal side, to the left extremity of the lower horizontal side, by a Slanting line.*

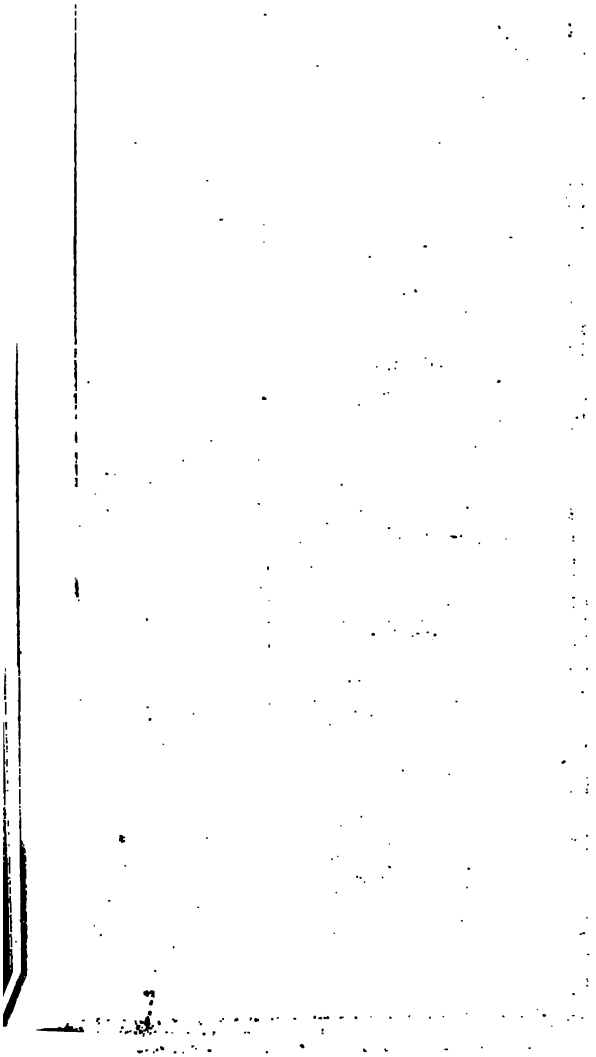
Twentieth line, for "*join the third,*" read, "*join the first,*" and after this sentence insert, "*Join the beginning of the upper horizontal line, to the end of the lower horizontal line.*"

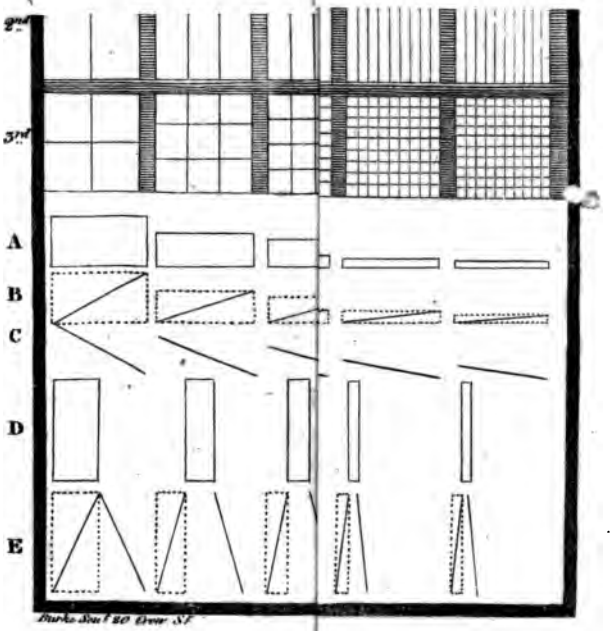
28, Tenth line, instead of "*Above the horizontal,*" read, "*Above the first horizontal,*" &c.

31, Third line, instead of by "*Upright,*" read, by "*two Upright.*"

Fourth from the bottom, Instead of the "*left extremity,*" read "*right extremity.*"

32, First line, Instead of "*right extremity,*" read, "*left extremity.*"
Fifth line, Instead of "*lesser,*" read, "*lower.*"

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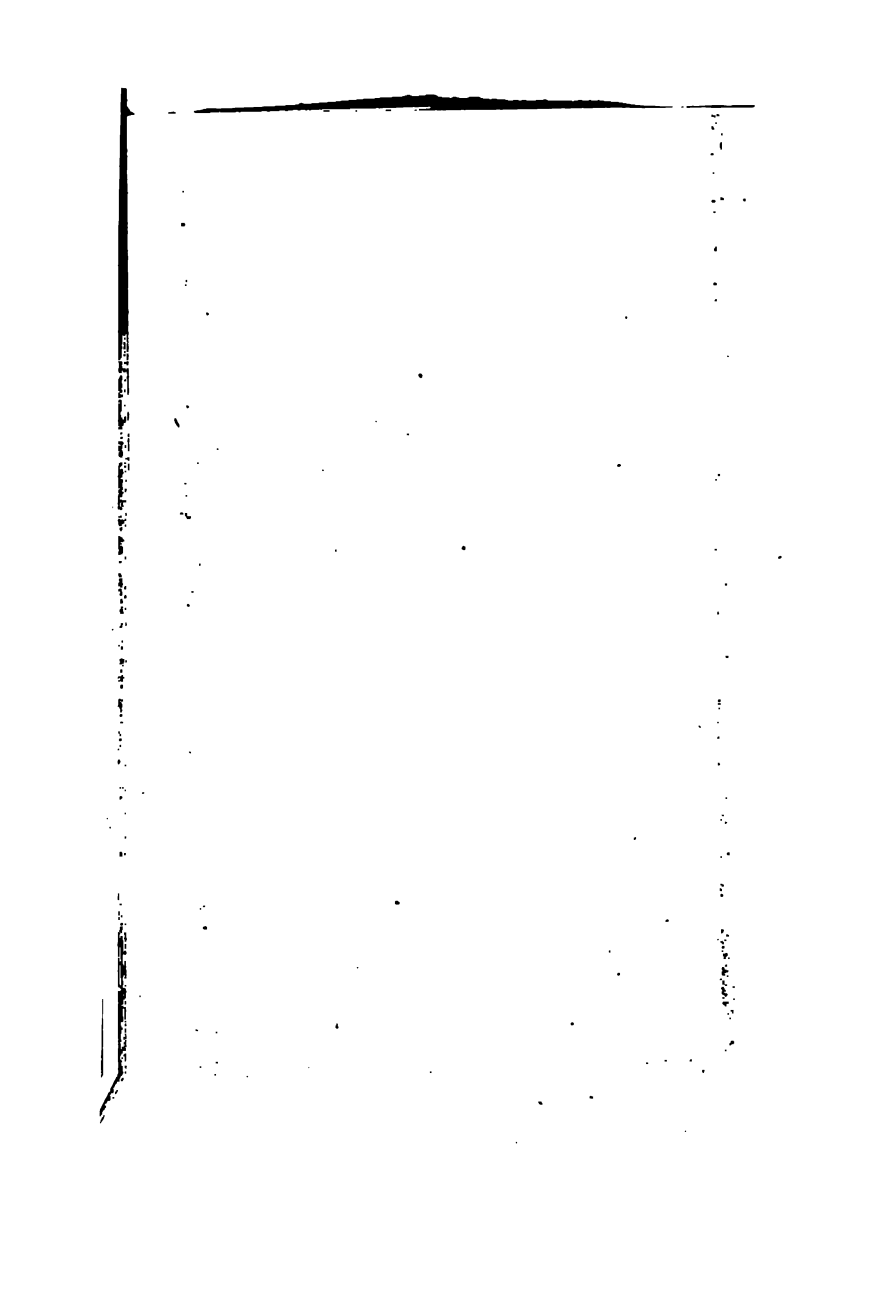

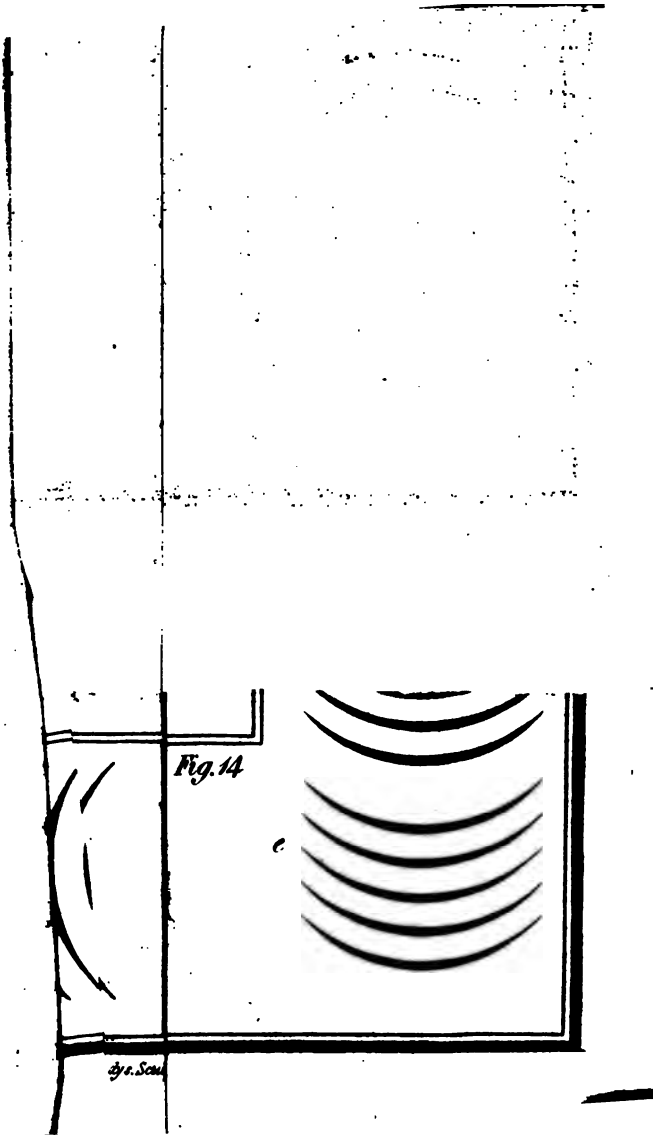
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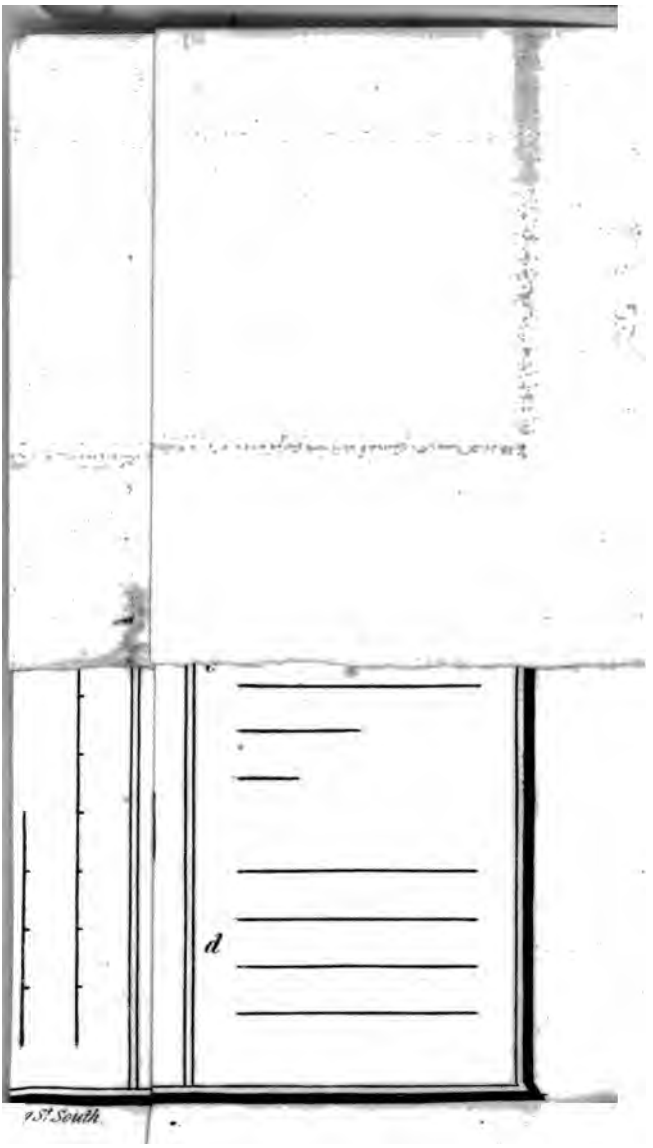
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